This section contains 30 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

1. Two forces P and Q, of magnitude 2F and 3F, respectively are at an angle \( \theta \) with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle \( \theta \) is:

   - (1) 30°
   - (2) 60°
   - (3) 90°
   - (4) 120°

   **Ans.** (4)

   **Sol.**

   \[ R = \sqrt{4F^2 + 9F^2 + 2 \times 2F \times 3F \cos\theta} \]

   \[ R = \sqrt{4F^2 + 9F^2 + 2 \times 2F \times 6F \cos\theta} \]

   \[ R = \sqrt{4F^2 + 9F^2 + 12F^2 \cos\theta} \]

   \[ R = \sqrt{13F^2 + 12F^2 \cos\theta} \]

   \[ \cos\theta = \frac{-12}{-24} = \frac{1}{2} \]

   \[ \theta = 60° \]

2. For the circuit shown below, the current through the Zener diode is:

   - (1) 14 mA
   - (2) 9 mA
   - (3) zero
   - (4) 5 mA

   **Ans.** (2)

   **Sol.**

   [Diagram of the circuit with annotations and calculations]
3. Two vectors $\mathbf{A}$ and $\mathbf{B}$ have equal magnitudes. The magnitude of $(\mathbf{A} + \mathbf{B})$ is $n$ times the magnitude of $(\mathbf{A} - \mathbf{B})$. The angle between $\mathbf{A}$ and $\mathbf{B}$ is:

$$\frac{\sin \left( \frac{n-1}{n+1} \right)}{\sin \left( \frac{n+1}{n} \right)} = \frac{\cos \left( \frac{n^2-1}{n^2+1} \right)}{\cos \left( \frac{n^2+1}{n^2-1} \right)}$$

Ans. \(n\)

Sol. $A^2 + A^2 + 2AB \cos \theta = n^2 (A^2 + A^2 - 2AB \cos \theta)$

$A^2 + A^2 + 2\sqrt{A^2} \cos \theta = n^2 (A^2 + A^2 - 2\sqrt{A^2} \cos \theta)$

$2A^2 (1 + \cos \theta) = 2A^2 n^2 (1 - \cos \theta)$

$1 + \cos \theta = n^2 - n^2 \cos \theta$

$\cos \theta = \frac{n^2 - 1}{n^2 + 1}$

$\theta = \cos^{-1} \left( \frac{n^2 - 1}{n^2 + 1} \right)$

4. The eye can be regarded as a single refracting surface. The radius of curvature of this surface is equal to that of corneas (7.8 mm). This surface separates two media of refractive indices $1$ and $1.34$. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus:

Let $d$ be the distance from the refracting surface. Then, $\frac{1}{f} = \frac{n_1 - n_2}{d}$

For $n_1 = 1$ and $n_2 = 1.34$, we have $\frac{1}{f} = 1.34 - 1 = 0.34$

Therefore, $f = \frac{1}{0.34} = 2.94$ mm

Ans. (4)

Sol. (4)

5. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be: (Assume that the highest frequency a person can hear is 20000 Hz)

A closed organ pipe produces overtones of $1.5 \times n\.\, kHz$, where $n$ is a positive integer.

For overtones to be distinctly heard, their frequencies must be below 20000 Hz.

Thus, $1.5 \times n \leq 20000$

$n = \frac{20000}{1.5}$

$n = 13333.33$

The number of overtones is 13333.

Ans. (4)
6. A current of 2 mA was passed through an unknown resistor which dissipated a power of 4.4 W. Dissipated power when an ideal power supply of 11 V is connected across it is:

\[ P = \frac{V^2}{R} \]

For second case

\[ P = \frac{(11)^2}{R} = 11 \times 10^{-5} \text{W} \]

**Solution**

\[ P = 4.4 = (2 \times 10^{-3}) V^2 \]

\[ R = 1.1 \times 10^6 \Omega \]

\[ (1) \ 1.1 \times 10^{-5} \text{W} \]

**Ans.** (4)

7. The diameter and height of a cylinder are measured by a meter scale to be 12.6 ± 0.1 cm and 34.2 ± 0.1 cm, respectively. What will be the value of its volume in appropriate significant figure?

**Solution**

\[ V = \frac{\pi d^2 h}{4} = 4260 \text{ cm}^3 \]

\[ \frac{\Delta V}{V} = \frac{2 \Delta d}{d} + \frac{\Delta h}{h} \]

\[ \Delta V = 2 \times 0.1V \times \frac{12.6}{12.9} + \frac{0.1V}{34.2} \]

\[ = 0.2 \times 0.342 \times 4260 \]

\[ = 80 \]

**Ans.** (3)

8. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time in second is:

**Solution**

\[ v = \pm \sqrt{5 - 4.25} = 3\text{ cm} \]

\[ a = \pm \sqrt{5 - 4.25} = 3\text{ cm/s} \]

\[ \omega = \frac{\frac{3}{4}}{T} \]

\[ T = \frac{8\pi}{3} \text{ sec} \]

**Ans.** (4)

9. At some location on earth the horizontal component of earth's magnetic field is 18 × 10^{-5} T. At this location, magnetic needle of length 0.12 m and pole strength 1.8 Am is suspended from its midpoint using a thread. It makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is:

**Solution**

The magnetic needle is suspended at an angle of 45° with the horizontal. The horizontal component of the Earth's magnetic field at this location is 18 × 10^{-5} T. The needle has a pole strength of 1.8 Am.

At equilibrium, the horizontal component of the needle's magnetic moment must balance the horizontal component of the Earth's magnetic field. The needle's magnetic moment is given by:

\[ \mu = 1.8 \text{ Am} \cdot \text{m} \]

The horizontal component of the needle's magnetic moment is:

\[ \mu_{\text{horizontal}} = \mu \cos(45°) \]

\[ = 1.8 \times 10^{-3} \text{ Am} \cdot \text{m} \times \cos(45°) \]

\[ = 0.126 \text{ Am} \cdot \text{m} \]

The horizontal component of the Earth's magnetic field is:

\[ B_{\text{horizontal}} = 18 \times 10^{-5} \text{ T} \]

For equilibrium, the horizontal component of the needle's magnetic moment must be equal to the horizontal component of the Earth's magnetic field:

\[ \mu_{\text{horizontal}} = B_{\text{horizontal}} \]

\[ 0.126 \text{ Am} \cdot \text{m} = 18 \times 10^{-5} \text{ T} \]

\[ \mu_{\text{horizontal}} = 1.8 \times 10^{-3} \text{ Am} \cdot \text{m} \]

However, this calculation does not directly determine the force that needs to be applied. To find the force, we need to consider the equilibrium and the geometry of the setup. The force applied should equilibrate the difference in forces due to the needle's inclination.

**Solution**

To find the required force, we consider the geometry and the forces acting on the needle. The force due to the Earth's magnetic field and the applied force must be equal and opposite for equilibrium.

\[ F = \frac{\mu_{\text{horizontal}}}{L} \]

\[ = \frac{1.8 \times 10^{-3} \text{ Am} \cdot \text{m}}{0.12 \text{ m}} \]

\[ = 15 \times 10^{-3} \text{ N} \]

However, this force is exerted at the midpoint of the needle, and the required force to be applied at one of its ends is the same magnitude but directed opposite to the horizontal component of the Earth's magnetic field.

**Ans.** The required force to be applied at one of the ends is 15 × 10^{-3} N, directed opposite to the horizontal component of the Earth's magnetic field.
10. A particle starts from origin at time \( t = 0 \) and moves along the positive \( x \)-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time \( t = 5s \)?

**Ans.** (1)

**Sol.**

\[
\frac{F}{2} = \frac{mB_0}{2} \times \frac{v}{v} = \frac{mB_0}{v}
\]

\[
F = \frac{2mvB_0}{v} = 3.6 \times 10^{-4} \text{ N}
\]

\[
= 6.5 \times 10^{-4} \text{ N}
\]

---

11. A parallel plate capacitor having capacitance 12 \( \mu \)F is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 8.5 is slipped between the plates. The work done by the capacitor on the slab is 12 \( \mu \)J. What is the change in potential energy of the capacitor due to the insertion of the dielectric slab?

**Ans.** (2)

**Sol.**

Area of each plate = \( \frac{1}{2} \times 2 \times 2 \) + \( \frac{1}{2} \times 3 \times 3 = 9 \text{ m}^2 \)

Displacement of charge = 2 + 4 + 3 = 9m

12. A particle which is experiencing a force, given by \( F = 3i - 12j \), undergoes a displacement of \( d = 4i \). If the particle had a kinetic energy of 3J at the beginning of the displacement, what is kinetic energy at the end of the displacement?

**Ans.** (2)

**Sol.**

\[
W = (U_f - U_i) = \frac{1}{2}C\varepsilon_0 \left( \left| \frac{q^2}{2C} \right| - \left| \frac{q^2}{2C} \right| \right) = \frac{1}{2} \left( \frac{k-1}{k} \right) \times 10^7 \times 12 \times 10^{-12} \times \frac{5.6}{6.6} = 500 \text{ J}
\]

By work energy theorem \( K_f = K_i + W \)

\[
12 \text{ J} = K_f - K_i \Rightarrow 12 + K_f = K_i
\]

\[
K_f = 12 + 3 = 15 \text{ J}
\]
13. Consider a Young's double slit experiment as shown in figure. What should be the slit separation \( d \) in terms of wavelength \( \lambda \) such that the first minima occurs directly in front of the slit \( S_1 \)?

![Diagram of Young's double slit experiment]

\( S_1 \) to \( S_2 \) = \( 2d \)

\( \Delta x = \frac{\lambda}{d} \)

\[ \frac{\lambda}{d} = \frac{\lambda}{2d} \]

**Ans.**

**Sol.**

\( \lambda = 2d \)

\( \lambda = \sqrt{5d} \)

\( \sqrt{5d} - 2d = \frac{\lambda}{2} \)

\( \frac{\lambda}{d} = \frac{\lambda}{2(\sqrt{5}-2)} \)

14. A cylindrical plastic bottle of negligible mass is filled with 310 ml of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency \( \omega \). The radius of the bottle is 2.5 cm and the density of water is \( 10^{3} \) kg/m³.

\( \omega = \frac{\sqrt{2g}}{r} \)

\[ \omega = \sqrt{\frac{2g}{r}} \]

**Ans.**

**Sol.**

\( \omega = 2.5 \times 10^{2} \times \sqrt{\frac{3.14 \times 10^{-2}}{3.10 \times 10^{-2}}} \)

\[ \omega = 2.5 \times 10^{2} \times \sqrt{0.1} \]

\[ \omega = 2.5 \times 10^{2} \times 0.1 \]

\[ \omega = 2.5 \times 10^{-1} \times 10^{-1} \]

15. An unknown metal of mass 192 g heated to a temperature of 100°C was immersed into a brass calorimeter of mass 128 g containing 240 g of water at a temperature of 8.4°C. Calculate the specific heat of the unknown metal if water temperature stabilizes at 21.5°C. (Specific heat of brass is 394 J kg⁻¹ K⁻¹)

\( m_{metal} = 192 \) g

\( m_{water} = 240 \) g

\( m_{calorimeter} = 128 \) g

\( T_{initial} = 100°C \)

\( T_{final} = 21.5°C \)

\( T_{water} = 8.4°C \)

\( c_{water} = 4.184 \) J/g°C

\[ c_{metal} = \frac{\Delta Q}{\Delta m} \]

\[ c_{metal} = \frac{m_{water} \cdot c_{water} \cdot (T_{final} - T_{initial})}{m_{metal}} \]

**Ans.**

\( c_{metal} = \frac{240 \times 4.184 \times (21.5 - 8.4)}{192} \)

\( c_{metal} = \frac{240 \times 4.184 \times 13.1}{192} \)

\( c_{metal} = \frac{1232 \times 13.1}{192} \)

\( c_{metal} = \frac{1232 \times 1.31}{192} \)

\( c_{metal} = \frac{1611.72}{192} \)

\( c_{metal} = 8.42 \) J/g°C
16. Two kg of a monatomic gas is at a pressure of $4 \times 10^5$ N/m². The density of the gas is $8$ kg/m³. What is the order of energy of the gas due to its thermal motion?

\[ \text{Ans.} \quad 10^3 \text{ J} \]  

17. Two stars of masses $3 \times 10^{30}$ kg each, and at distance $2 \times 10^{11}$ m rotate in a plane about their common centre of mass. O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is \((\text{Take Gravitational constant } G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{kg}^{-2})\)

What is the order of energy of the gas due to its thermal motion?

\[ \begin{align*}
\text{Ans.} & \quad 10^3 \text{ J} \\
& \quad 10^4 \text{ J} \\
& \quad 10^5 \text{ J} \\
& \quad 10^6 \text{ J}
\end{align*} \]

18. The half-life of an ideal monatomic gas is heated at constant pressure of 1 atm from 20°C to 90°C.

\[ \text{Ans.} \quad 10^{11} \text{ m}^3 \\
& \quad 10^{12} \text{ m}^3 \\
& \quad 10^{13} \text{ m}^3 \\
& \quad 10^{14} \text{ m}^3
\]
20. Four equal point charges \( Q \) each are placed in the xy-plane at \((0, 2), (4, 2), (4, -2)\) and \((0, -2)\). The work required to put a fifth charge \( Q \) at the origin of the coordinate system will be

\[
W = \sum \left( \frac{Q^2}{4\pi\varepsilon_0} \cdot \frac{1}{r} \right)
\]

(1) \( \frac{Q^2}{4\pi\varepsilon_0} \cdot \frac{1}{r} \)

(2) \( \frac{Q^2}{4\pi\varepsilon_0} \cdot \frac{1}{\sqrt{r}} \)

(3) \( \frac{Q^2}{4\pi\varepsilon_0} \cdot \frac{1}{\sqrt{r}} \)

(4) \( \frac{Q^2}{2\sqrt{2\pi\varepsilon_0}} \)

Ans. (2)

Sol.

\[
W_{\text{tot}} = UV - U_1 - U_2
\]

\[
= 0 - \frac{kQ^2}{2} - \frac{kQ^2}{2} + \frac{kQ^2}{2} + \frac{kQ^2}{2}
\]

\[
= \frac{kQ^2}{2}
\]

\[
W_{\text{tot}} = \frac{1}{2}kQ^2
\]

21. A metal plate of area \( 1 \times 10^{-4} \) m\(^2\) is illuminated by a radiation of intensity 16 mW/m\(^2\). The work function of the metal is 5 eV. The energy of the incident photons is 10 eV and only 10% of its produces photoelectrons. The number of emitted photoelectrons per second and their maximum energy, respectively, will be

\[
1 \times 10^{-4} \text{ m}^2 \text{ plate}\]

\[
16 \text{ mW/m}^2 \text{ intensity}
\]

\[
5 \text{ eV work function}
\]

\[
10 \text{ eV energy of photons}
\]

\[
10\% \text{ efficiency}
\]

\[
\text{No. of photons incident per unit time}
\]

\[
\text{No. of photoelectrons ejected per unit time}
\]

\[
\text{Maximum kinetic energy}
\]

\[
\text{Number of photoelectrons ejected per second}
\]

\[
\text{Energy of photoelectrons ejected per second}
\]

Ans. (2)

Sol.

Maximum kinetic energy \( K_{\text{max}} = E - \phi \)

\[
K_{\text{max}} = 5 \text{ eV}
\]

\[
10^{11} \text{ photons per second}
\]

\[
10^{10} \text{ photoelectrons per second}
\]

\[
10^9 \text{ photons}
\]

\[
10^{10} \text{ photoelectrons}
\]

\[
10^{10}\text{ photons}
\]

\[
10^{10}\text{ photoelectrons}
\]
22. The self induced emf of a coil is 25 volts. When the current in it is changed at uniform rate from 10A to 25A in 1s, the change in the energy of the inductance is:

\[ 25 = L \times \frac{(25 - 10)}{1} \]

\[ L = \frac{5}{3} \text{ H} \]

energy of inductor \( E = \frac{1}{2} L I^2 \)

\[ \text{ energy of } \text{ coil } E = \frac{1}{2} \times \frac{5}{3} \times (25 - 10) \]

\[ \Rightarrow \Delta E = \frac{1}{2} \times \frac{5}{3} \times 125 = 437.5 \text{ J} \]

Ans. (4) 437.5 J

23. Two identical spherical balls of mass \( M \) and radius \( R \) each are stuck on two ends of a rod of length \( 2R \) and mass \( M \) (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is:

\[ I = \frac{2}{5} M R^2 + M (2R)^2 \]

\[ I = \frac{209}{15} M R^2 \]

Ans. (3) \( \frac{109}{15} M R^2 \)

24. Charge \(-q\) and \(+q\) located at A and B respectively, constitute an electric dipole. Distance \( AB = 2a \), O is the mid-point of the dipole and OP is perpendicular to AB. A charge \( Q \) is placed at P where OP = \( y \) and \( y >> 2a \). The charge \( Q \) experiences an electrostatic force \( F \). If \( Q \) is now moved along the equatorial line to \( P' \) such that \( OP = \frac{y}{3} \), the force on \( Q \) will be close to:

\[ \frac{y}{3} >> 2a \]

A and B are three points. Charge \(-q\) and \(+q\) are two closed conductors. Distance \( AB = 2a \) is between \( AB \) and point \( O \). OP \( \perp \) AB and \( OP = y \) is such that \( y >> 2a \). When \( Q \) is placed at any point \( P \) on \( OP \) then the force on \( Q \) is close to:

\[ \frac{y}{3} >> 2a \]

Ans. (2) \( \frac{2F}{3} \)
25. The electric field of a plane polarized electromagnetic wave in free space at time \( t = 0 \) is given by an expression \( E(x, y, z) = 10 \cos(6x + 8z) \). The magnetic field \( B(x, y, z) \) is given by: (c is the velocity of light)

\[
E(x, y, z) = 10 \cos(6x + 8z)
\]

\[
B(x, y, z) = \frac{\mathbf{c}}{c} \times (\mathbf{c} \times \mathbf{E})
\]

\[
= \frac{1}{c} \left( \mathbf{6} \mathbf{x} - \mathbf{8} \mathbf{z} \right) \cos(6x + 8z + 10ct)
\]

\[
= \frac{1}{c} \left( \mathbf{6} \mathbf{x} + \mathbf{8} \mathbf{z} \right) \cos(6x + 8z - 10ct)
\]

Answ. (4)

Sol. \( E = \frac{E_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{r})}{c} \)

- At any time \( t \):
  \[ k = 6\mathbf{i} + 8\mathbf{k} \]
  \[ \mathbf{E} = 10\cos(6x + 8z - 10ct) \]

\[ j = 10 \]

26. The modulation frequency of an AM radio station is 250 kHz, which is 10% of the carrier wave. If another AM station approaches you for licence what broadcast frequency will you allot? (Assuming that the broadcast frequency is \( 1000 \) kHz)

\[ \lambda = \frac{2\pi c}{f} \]

\[ \lambda = \frac{2\pi (c - v)}{f} \]

\[ \lambda = \frac{2\pi c}{f} = \frac{2\pi c}{0.2f} \]

\[ \lambda = \frac{2\pi c}{0.2f} = \frac{2\pi c}{0.2 \times 10^6} = 10^4 \]

Comparing the coefficient on both sides:

\[ B_0 = 0.6 \]

\[ B_0 = 0.8 \]

\[ B_1 = \sqrt{10^4 - 0.6^2} = 10^2 \]

\[ B_2 = \sqrt{10^4 - 0.8^2} = 10^2 \]

So magnetic field \( B = \frac{1}{c} \cdot \mathbf{6} \mathbf{x} + 6\mathbf{k} \) Tesla

27. The Wheatstone bridge shown in the figure here, gets balanced when the carbon resistor used as \( R_3 \) has the colour code (Orange, Red, Brown). The resistors \( R_2 \) and \( R_4 \) are \( 800 \) and \( 400 \), respectively. Assuming that the colour code for the carbon resistors gives their accurate values, the colour code for the carbon resistor, used as \( R_3 \), would be:

\[ R_3 \]

\[ R_3 \]

\[ R_3 \]

\[ R_3 \]

\[ R_3 \]

The following options are possible for the carbon resistors:

- Red, Black, Brown
- Grey, Black, Brown
- Brown, Blue, Black
- Brown, Blue, Black

Assuming that the colour code for the carbon resistors gives their accurate values, the colour code for the carbon resistor, used as \( R_3 \), would be:

- Red, Black, Brown
- Grey, Black, Brown
- Brown, Blue, Black
- Brown, Blue, Black

Answ. (4)
28. A hoop and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder at $T_h$ and $T_c$ respectively, then

The magnetic dipole moments of the two objects can be given by $\mu_i = I_i a_i$, where $I_i$ is the magnetic moment and $a_i$ is the moment arm. The frequency of oscillation is given by $f = \frac{1}{2\pi} \frac{1}{T}$. Hence, $\omega = 2\pi f = \frac{2\pi}{T}$. The magnetic moment of the hoop is twice that of the cylinder, thus $I_h = 2I_c$. Using the formula for the oscillation period $T = 2\pi \sqrt{\frac{I}{MB}}$, we get:

$T_h = 2\pi \sqrt{\frac{2I_c}{MB}} = \sqrt{2} T_c$

29. A rigid massless rod of length $3\ell$ has two masses attached at both ends as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be:

The moment of inertia of the system is $I = M_1(\ell/2)^2 + M_2(\ell/2)^2 + M_3(\ell)^2 = \frac{5}{2}ML^2$. The torque is $\tau = -M_3g(\ell - 2\ell) = -5Mg\ell$. The angular acceleration is $\alpha = \frac{\tau}{I} = \frac{-5Mg\ell}{\frac{5}{2}ML^2} = -\frac{2g}{L}$. Therefore, the angular acceleration is $\frac{2}{3\ell}$.

30. The actual value of resistance $R$, shown in the figure is 30Ω. This is measured in an experiment as shown using the standard formula $R = \frac{V}{I}$, where $V$ and $I$ are the readings of the voltmeter and ammeter, respectively. If the measured value of $R$ is 5% less, then the internal resistance of the voltmeter is:

The internal resistance of the voltmeter is given by $r = \frac{V}{I} - R$. Here, $V = \frac{3}{4}V$ and $I = \frac{1}{4}I$. Therefore, $r = \frac{3}{4}V - \frac{30}{\frac{1}{4}I} = \frac{3V}{I} - 30$.
Ans. (3)

Sol.

Let the measured voltage be $V_m$ and

$m$ be the ratio of the measured $V_m$ to the

Let the measured current be $i_m$ and

$m$ be the ratio of the measured $i_m$ to the

Let the ammeter be ideal, thus

the current measured by the ammeter

$V_m = R_m \cdot i_m$

$\frac{V_m}{i_m} = R_m$

$\frac{R}{R + R_m} = \frac{1}{R_m}$

$\frac{1}{R_m} = \frac{1}{R} + \frac{1}{R_m}$

$\frac{1}{R_m} = \frac{1}{R} - 0.95$

$R_m = 19 \Omega$

$R = 570 \Omega$