FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Monday 08th APRIL, 2019) TIME : 9 : 30 AM To 12 : 30 PM

MATHEMATICS

TEST PAPER WITH ANSWER

1. The shortest distance between the line y = x and the curve $y^2 = x - 2$ is :

(1) $\frac{7}{4\sqrt{2}}$ (2) $\frac{7}{8}$

(3) $\frac{11}{4\sqrt{2}}$ (4) 2

Official Ans. by NTA (1)

2. Let y = y(x) be the solution of the differential equation, $\left(x^2+1\right)^2\frac{dy}{dx}+2x\left(x^2+1\right)y=1$ such that y(0) = 0. If $\sqrt{a}y(1)=\frac{\pi}{32}$, then the value of 'a' is :

(1) $\frac{1}{2}$ (2) $\frac{1}{16}$

(3) $\frac{1}{4}$ (4) 1

Official Ans. by NTA (2)

3. A point on the straight line, 3x + 5y = 15 which is equidistant from the coordinate axes will lie only in :

(1) 1st and 2nd quadrants

(2) 4th quadrant

(3) 1st, 2nd and 4th quadrant

(4) 1st quadrant

Official Ans. by NTA (1)

4. If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is :

(1) 2 (2) 3

(3) 4 (4) 5

Official Ans. by NTA (3)

5. $\lim\limits_{x\to 0}\frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$ equals :

(1) $2\sqrt{2}$ (2) $4\sqrt{2}$

(3) $\sqrt{2}$ (4) 4

Official Ans. by NTA (2)

6. The length of the perpendicular from the point (2, –1, 4) on the straight line, $\frac{x+3}{10}=\frac{y-2}{-7}=\frac{z}{1}$ is :

(1) less than 2

(2) greater than 3 but less than 4

(3) greater than 4

(4) greater than 2 but less than 3

Official Ans. by NTA (2)

7. The magnitude of the projection of the vector $2\hat{i}+3\hat{j}+\hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$, is :

(1) $\frac{\sqrt{3}}{2}$ (2) $\sqrt{\frac{3}{2}}$

(3) $\sqrt{6}$ (4) $3\sqrt{6}$

Official Ans. by NTA (2)

8. The contrapositive of the statement "If you are born in India, then you are a citizen of India", is :

(1) If you are born in India, then you are not a citizen of India.

(2) If you are not a citizen of India, then you are not born in India.

(3) If you are a citizen of India, then you are born in India.

(4) If you are not born in India, then you are not a citizen of India.

Official Ans. by NTA (2)

9. The mean and variance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the product of the remaining two observations is :

(1) 40 (2) 49

(3) 48 (4) 45

Official Ans. by NTA (3)

10. If $f(x)=\frac{2-x\cos x}{2+x\cos x}$ and $g(x)=\log_e x, (x>0)$ then the value of integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} g(f(x))\,dx$ is :

(1) $\log_e 3$ (2) $\log_e 2$
(3) $\log_e e$ (4) $\log_e 1$

Official Ans. by NTA (4)

11. If the tangents on the ellipse $4x^2 + y^2 = 8$ at the points (1, 2) and (a, b) are perpendicular to each other, then a^2 is equal to :

(1) $\frac{64}{17}$ (2) $\frac{2}{17}$
(3) $\frac{128}{17}$ (4) $\frac{4}{17}$

Official Ans. by NTA (2)

12. If $\alpha=\cos^{-1}\left(\frac{3}{5}\right), \beta=\tan^{-1}\left(\frac{1}{3}\right)$, where $0<\alpha,\beta<\frac{\pi}{2}$, then $\alpha - \beta$ is equal to :

(1) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (2) $\tan^{-1}\left(\frac{9}{14}\right)$
(3) $\cos^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (4) $\tan^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

Official Ans. by NTA (1)

13. If S_1 and S_2 are respectively the sets of local minimum and local maximum points of the function, $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$, $x \in R$, then :

(1) $S_1 = \{-2, 1\}; S_2 = \{0\}$
(2) $S_1 = \{-2, 0\}; S_2 = \{1\}$
(3) $S_1 = \{-2\}; S_2 = \{0, 1\}$
(4) $S_1 = \{-1\}; S_2 = \{0, 2\}$

Official Ans. by NTA (1)

14. Let O(0, 0) and A(0, 1) be two fixed points. Then the locus of a point P such that the perimeter of ΔAOP is 4, is :

(1) $8x^2 - 9y^2 + 9y = 18$
(2) $9x^2 + 8y^2 - 8y = 16$
(3) $8x^2 + 9y^2 - 9y = 18$
(4) $9x^2 - 8y^2 + 8y = 16$

Official Ans. by NTA (2)

15. Let $A=\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}, (\alpha\in R)$ such that $A^{32}=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$. Then a value of α is

(1) $\frac{\pi}{16}$ (2) 0
(3) $\frac{\pi}{32}$ (4) $\frac{\pi}{64}$

Official Ans. by NTA (4)

16. If $f(x)=\log_e\left(\frac{1-x}{1+x}\right), |x|<1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to :

(1) $2f(x)$ (2) $2f(x^2)$
(3) $(f(x))^2$ (4) $-2f(x)$

Official Ans. by NTA (1)

17. The equation of a plane containing the line of intersection of the planes $2x - y - 4 = 0$ and $y + 2z - 4 = 0$ and passing through the point (1, 1, 0) is :

(1) $x + 3y + z = 4$ (2) $x - y - z = 0$
(3) $x - 3y - 2z = -2$ (4) $2x - z = 2$

Official Ans. by NTA (2)

18. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. $(91, n) > 1$ is :

(1) 3221 (2) 3121
(3) 3203 (4) 3303

Official Ans. by NTA (2)

19. The sum of the series $2.{}^{20}C_0 + 5.{}^{20}C_1 + 8.{}^{20}C_2 + 11.{}^{20}C_3 + ... + 62.{}^{20}C_{20}$ is equal to :

(1) 2^{24} (2) 2^{25}
(3) 2^{26} (4) 2^{23}

Official Ans. by NTA (2)

20. The sum of the solutions of the equation $\left|\sqrt{x}-2\right|+\sqrt{x}\left(\sqrt{x}-4\right)+2=0, (x>0)$ is equal to :

(1) 4 (2) 9
(3) 10 (4) 12

Official Ans. by NTA (3)

21. Let A and B be two non-null events such that $A \subset B$. Then, which of the following statements is always correct ?

(1) $P(A|B) = 1$

(2) $P(A|B) = P(B) - P(A)$

(3) $P(A|B) \leq P(A)$

(4) $P(A|B) \geq P(A)$

Official Ans. by NTA (4)

22. The sum of the co-efficients of all even degree terms in x in the expansion of $\left(x+\sqrt{x^3-1}\right)^6+\left(x-\sqrt{x^3-1}\right)^6, (x>1)$ is equal to :

(1) 32 (2) 26

(3) 29 (4) 24

Official Ans. by NTA (4)

23. The area (in sq. units) of the region $A = \{(x, y) \in R \times R | 0 \leq x \leq 3, 0 \leq y \leq 4, y \leq x^2 + 3x\}$ is :

(1) $\frac{53}{6}$ (2) $\frac{59}{6}$

(3) 8 (4) $\frac{26}{3}$

Official Ans. by NTA (2)

24. Let $f : [0, 2] \to R$ be a twice differentiable function such that $f''(x) > 0$, for all $x \in (0, 2)$. If $\phi(x) = f(x) + f(2 - x)$, then ϕ is :

(1) decreasing on (0, 2)

(2) decreasing on (0, 1) and increasing on (1, 2)

(3) increasing on (0, 2)

(4) increasing on (0, 1) and decreasing on (1, 2)

Official Ans. by NTA (2)

25. The sum of the squares of the lengths of the chords intercepted on the circle, $x^2 + y^2 = 16$, by the lines, $x + y = n$, $n \in N$, where N is the set of all natural numbers, is :

(1) 320 (2) 160

(3) 105 (4) 210

Official Ans. by NTA (4)

26. All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is :

(1) 175 (2) 162

(3) 160 (4) 180

Official Ans. by NTA (4)

27. $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx$ is equal to :

(where c is a constant of integration)

(1) $2x + \sin x + 2\sin 2x + c$

(2) $x + 2\sin x + 2\sin 2x + c$

(3) $x + 2\sin x + \sin 2x + c$

(4) $2x + \sin x + \sin 2x + c$

Official Ans. by NTA (3)

28. If $2y = \left(\cot^{-1}\left(\frac{\sqrt{3}\cos x + \sin x}{\cos x - \sqrt{3}\sin x}\right)\right)^2, x \in \left(0, \frac{\pi}{2}\right)$,

then $\frac{dy}{dx}$ is equal to :

(1) $2x - \frac{\pi}{3}$ (2) $\frac{\pi}{3} - x$

(3) $\frac{\pi}{6} - x$ (4) $x - \frac{\pi}{6}$

Official Ans. by NTA (4)

29. The greatest value of $c \in R$ for which the system of linear equations

$x - cy - cz = 0$

$cx - y + cz = 0$

$cx + cy - z = 0$

has a non-trivial solution, is :

(1) $\frac{1}{2}$ (2) –1

(3) 0 (4) 2

Official Ans. by NTA (1)

30. If $\cos(\alpha+\beta)=\frac{3}{5}, \sin(\alpha-\beta)=\frac{5}{13}$ and

$0 < \alpha, \beta < \frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to :

(1) $\frac{21}{16}$ (2) $\frac{63}{52}$

(3) $\frac{33}{52}$ (4) $\frac{63}{16}$

Official Ans. by NTA (4)