FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Wednesday 10th APRIL, 2019) TIME : 9 : 30 AM To 12 : 30 PM

MATHEMATICS

TEST PAPER WITH ANSWER

1. If for some $x \in R$, the frequency distribution of the marks obtained by 20 students in a test is :

Marks	2	3	5	7
Frequencey	$(x+1)^2$	$2x-5$	x^2-3x	x

then the mean of the marks is :

(1) 2.8 (2) 3.2 (3) 3.0 (4) 2.5

Official Ans. by NTA (1)

2. If $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ and

$\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}$, $x \neq 0$; then for all $\theta \in \left(0, \frac{\pi}{2}\right)$:

(1) $\Delta_1 - \Delta_2 = x(\cos 2\theta - \cos 4\theta)$

(2) $\Delta_1 + \Delta_2 = -2x^3$

(3) $\Delta_1 - \Delta_2 = -2x^3$

(4) $\Delta_1 + \Delta_2 = -2(x^3 + x - 1)$

Official Ans. by NTA (2)

3. If $\lim\limits_{x\to 1} \frac{x^4-1}{x-1} = \lim\limits_{x\to k} \frac{x^3-k^3}{x^2-k^2}$, then k is :

(1) $\frac{3}{8}$ (2) $\frac{3}{2}$ (3) $\frac{4}{3}$ (4) $\frac{8}{3}$

Official Ans. by NTA (4)

4. If the system of linear equations

$x + y + z = 5$

$x + 2y + 2z = 6$

$x + 3y + \lambda z = \mu$, $(\lambda, \mu \in R)$, has infinitely many solutions, then the value of $\lambda + \mu$ is :

(1) 12 (2) 10 (3) 9 (4) 7

Official Ans. by NTA (2)

5. If the circles $x^2 + y^2 + 5Kx + 2y + K = 0$ and $2(x^2 + y^2) + 2Kx + 3y - 1 = 0$, $(K \in R)$, intersect at the points P and Q, then the line $4x + 5y - K = 0$ passes through P and Q for :

(1) exactly two values of K

(2) exactly one value of K

(3) no value of K.

(4) infinitely many values of K

Official Ans. by NTA (3)

6. Le $f(x) = x^2$, $x \in R$. For any $A \subseteq R$, define $g(A) = \{x \in R, f(x) \in A\}$. If $S = [0, 4]$, then which one of the following statements is not true ?

(1) $f(g(S)) \neq f(S)$ (2) $f(g(S)) = S$

(3) $g(f(S)) = g(S)$ (4) $g(f(S)) \neq S$

Official Ans. by NTA (3)

7. Let $f(x) = e^x - x$ and $g(x) = x^2 - x$, $\forall x \in R$. Then the set of all $x \in R$, where the function $h(x) = (f \circ g)(x)$ is increasing, is :

(1) $\left[-1, \frac{-1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$ (2) $\left[0, \frac{1}{2}\right] \cup [1, \infty)$

(3) $\left[\frac{-1}{2}, 0\right] \cup [1, \infty)$ (4) $[0, \infty)$

Official Ans. by NTA (2)

8. Which one of the following Boolean expressions is a tautology ?

(1) $(P \vee q) \wedge (\sim p \vee \sim q)$ (2) $(P \wedge q) \vee (p \wedge \sim q)$

(3) $(P \vee q) \wedge (p \vee \sim q)$ (4) $(P \vee q) \vee (p \vee \sim q)$

Official Ans. by NTA (4)

9. All the pairs (x, y) that satisfy the inequality

$2\sqrt{\sin^2 x - 2\sin x + 5} \cdot \frac{1}{4^{\sin^2 y}} \leq 1$ also satisfy the eauation.

(1) $\sin x = |\sin y|$ (2) $\sin x = 2 \sin y$

(3) $2|\sin x| = 3\sin y$ (4) $2\sin x = \sin y$

Official Ans. by NTA (1)

10. The number of 6 digit numbers that can be formed using the digits 0, 1, 2, 5, 7 and 9 which are divisible by 11 and no digit is repeated, is :

(1) 36 (2) 60 (3) 48 (4) 72

Official Ans. by NTA (2)

11. Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls is :

(1) $\frac{1}{11}$ (2) $\frac{1}{17}$ (3) $\frac{1}{10}$ (4) $\frac{1}{12}$

Official Ans. by NTA (1)

12. The sum

$$\frac{3\times1^3}{1^2}+\frac{5\times\left(1^3+2^3\right)}{1^2+2^2}+\frac{7\times\left(1^3+2^3+3^3\right)}{1^2+2^2+3^2}+.....$$

(1) 660 (2) 620 (3) 680 (4) 600

Official Ans. by NTA (1)

13. If a directrix of a hyperbola centred at the origin and passing through the point $\left(4,-2\sqrt{3}\right)$ is $5x = 4\sqrt{5}$ and its eccentricity is e, then :

(1) $4e^4 - 24e^2 + 35 = 0$

(2) $4e^4 + 8e^2 - 35 = 0$

(3) $4e^4 - 12e^2 - 27 = 0$

(4) $4e^4 - 24e^2 + 27 = 0$

Official Ans. by NTA (1)

14. If $f(x) = \begin{cases} \frac{\sin(p+1)+\sin x}{x}, & x<0 \\ q, & x=0 \\ \frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}, & x>0 \end{cases}$

is continuous at x = 0, then the ordered pair (p,q) is equal to :

(1) $\left(\frac{5}{2},\frac{1}{2}\right)$ (2) $\left(-\frac{3}{2},-\frac{1}{2}\right)$

(3) $\left(-\frac{1}{2},\frac{3}{2}\right)$ (4) $\left(-\frac{3}{2},\frac{1}{2}\right)$

Official Ans. by NTA (4)

15. If y = y (x) is the solution of the differential equation

$\frac{dy}{dx} = (\tan x - y)\sec^2 x$, $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, such that $y(0) = 0$, then $y\left(-\frac{\pi}{4}\right)$ is equal to :

(1) $2+\frac{1}{e}$ (2) $\frac{1}{2}-e$ (3) $e - 2$ (4) $\frac{1}{2}-e$

Official Ans. by NTA (3)

16. If the line $x - 2y = 12$ is tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $\left(3,\frac{-9}{2}\right)$, then the length of the latus recturm of the ellipse is :

(1) 9 (2) $8\sqrt{3}$ (3) $12\sqrt{2}$ (4) 5

Official Ans. by NTA (1)

17. The value of $\int_0^{2\pi}\left[\sin 2x(1+\cos 3x)\right]dx$, where [t] denotes the greatest integer function, is :

(1) -2π (2) π (3) $-\pi$ (4) 2π

Official Ans. by NTA (3)

18. The region represented by $|x-y| \le 2$ and $|x+y| \le 2$ is bounded by a :

(1) square of side length $2\sqrt{2}$ units

(2) rhombus of side length 2 units

(3) square of area 16 sq, units

(4) rhombus of area $8\sqrt{2}$ sq. units

Official Ans. by NTA (1)

19. The line x = y touches a circle at the point (1, 1). If the circle also passes through the point (1, –3), then its radius is :

(1) $3\sqrt{2}$ (2) 3 (3) $2\sqrt{2}$ (4) 2

Official Ans. by NTA (1)

20. Let A(3, 0, –1), B (2, 10, 6) and C(1, 2, 1) be the vertices of a triangle and M be the midpoint of AC. If G divides BM in the ratio, 2 : 1, then $\cos\left(\angle GOA\right)$ (O being the origin) is equal to :

(1) $\frac{1}{\sqrt{30}}$ (2) $\frac{1}{6\sqrt{10}}$

(3) $\frac{1}{\sqrt{15}}$ (4) $\frac{1}{2\sqrt{15}}$

Official Ans. by NTA (3)

21. Let $f : R \to R$ be differentiable at $c \in R$ and $f(c) = 0$. If $g(x) = |f(x)|$, then at $x = c$, g is :

(1) differentiable if $f'(c) = 0$

(2) not differentiable

(3) differentiable if $f'(c) \neq 0$

(4) not differentiable if $f'(c) = 0$

Official Ans. by NTA (1)

22. If α and β are the roots of the quadratic equation, $x^2 + x\sin\theta - 2\sin\theta = 0$, $\theta \in \left(0, \frac{\pi}{2}\right)$, then $\frac{\alpha^{12} + \beta^{12}}{\left(\alpha^{-12} + \beta^{-12}\right)(\alpha - \beta)^{24}}$ is equal to :

(1) $\frac{2^6}{(\sin\theta + 8)^{12}}$ (2) $\frac{2^{12}}{(\sin\theta - 8)^6}$

(3) $\frac{2^{12}}{(\sin\theta - 4)^{12}}$ (4) $\frac{2^{12}}{(\sin\theta + 8)^{12}}$

Official Ans. by NTA (4)

23. If the length of the perpendicular from the point $(\beta, 0, \beta)$ $(\beta \neq 0)$ to the line, $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to :

(1) -1 (2) 2 (3) -2 (4) 1

Official Ans. by NTA (1)

24. If $\int \frac{dx}{\left(x^2 - 2x + 10\right)^2}$

$= A\left(\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{f(x)}{x^2 - 2x + 10}\right) + C$

where C is a constant of integration, then :

(1) $A = \frac{1}{27}$ and $f(x) = 9(x - 1)$

(2) $A = \frac{1}{81}$ and $f(x) = 3(x - 1)$

(3) $A = \frac{1}{54}$ and $f(x) = 9(x - 1)^2$

(4) $A = \frac{1}{54}$ and $f(x) = 3(x - 1)$

Official Ans. by NTA (4)

25. ABC is a triangular park with AB = AC = 100 metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}\left(3\sqrt{2}\right)$ and $\text{cosec}^{-1}\left(2\sqrt{2}\right)$ respectively, then the height of the tower (in metres) is :

(1) $10\sqrt{5}$ (2) $\frac{100}{3\sqrt{3}}$ (3) 20 (4) 25

Official Ans. by NTA (3)

26. If $a_1, a_2, a_3, \ldots\ldots, a_n$ are in A.P. and $a_1 + a_4 + a_7 + \ldots\ldots + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to :

(1) 38 (2) 98 (3) 76 (4) 64

Official Ans. by NTA (3)

27. $\lim\limits_{n\to\infty}\left(\frac{(n+1)^{1/3}}{n^{4/3}} + \frac{(n+2)^{1/3}}{n^{4/3}} + \ldots\ldots + \frac{(2n)^{1/3}}{n^{4/3}}\right)$ is equal to :

(1) $\frac{4}{3}(2)^{4/3}$ (2) $\frac{3}{4}(2)^{4/3} - \frac{4}{3}$

(3) $\frac{3}{4}(2)^{4/3} - \frac{3}{4}$ (4) $\frac{4}{3}(2)^{3/4}$

Official Ans. by NTA (3)

28. If Q(0, –1, –3) is the image of the point P in the plane $3x - y + 4z = 2$ and R is the point (3, –1, –2), then the area (in sq. units) of ΔPQR is :

(1) $\frac{\sqrt{65}}{2}$ (2) $\frac{\sqrt{91}}{4}$ (3) $2\sqrt{13}$ (4) $\frac{\sqrt{91}}{2}$

Official Ans. by NTA (4)

29. If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1 + ax + bx^2)(1 - 3x)^{15}$ in powers of x, then the ordered pair (a, b) is equal to :

(1) (28, 315) (2) (–54, 315)

(3) (–21, 714) (4) (24, 861)

Official Ans. by NTA (1)

30. If $a > 0$ and $z = \frac{(1+i)^2}{a - i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\bar{z}$ is equal to :

(1) $-\frac{3}{5} - \frac{1}{5}i$ (2) $-\frac{1}{5} + \frac{3}{5}i$

(3) $-\frac{1}{5} - \frac{3}{5}i$ (4) $\frac{1}{5} - \frac{3}{5}i$

Official Ans. by NTA (3)