FINAL JEE–MAIN EXAMINATION – APRIL,2019

(Held On Wednesday 10th APRIL, 2019) TIME : 2 : 30 PM To 5 : 30 PM

MATHEMATICS

TEST PAPER WITH ANSWER

1. The distance of the point having position vector $-\hat{i}+2\hat{j}+6\hat{k}$ from the straight line passing through the point (2, 3, –4) and parallel to the vector, $6\hat{i}+3\hat{j}-4\hat{k}$ is :

(1) 7 (2) $4\sqrt{3}$

(3) $2\sqrt{13}$ (4) 6

Official Ans. by NTA (1)

2. If both the mean and the standard deviation of 50 observations $x_1, x_2,....,x_{50}$ are equal to 16, then the mean of $(x_1 - 4)^2$, $(x_2 - 4)^2$,..... $(x_{50} - 4)^2$ is :

(1) 525 (2) 380

(3) 480 (4) 400

Official Ans. by NTA (4)

3. A perpendicular is drawn from a point on the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{1}$ to the plane x + y + z = 3 such that the foot of the perpendicular Q also lies on the plane x – y + z = 3. Then the co-ordinates of Q are :

(1) (2, 0, 1) (2) (4, 0, –1)

(3) (–1, 0, 4) (4) (1, 0, 2)

Official Ans. by NTA (1)

4. The tangent and normal to the ellipse $3x^2 + 5y^2 = 32$ at the point P(2, 2) meet the x-axis at Q and R, respectively. Then the area (in sq. units) of the triangle PQR is :

(1) $\frac{14}{3}$ (2) $\frac{16}{3}$

(3) $\frac{68}{15}$ (4) $\frac{34}{15}$

Official Ans. by NTA (3)

5. Let λ be a real number for which the system of linear equations

x + y + z = 6

4x + λy – λz = λ – 2

3x + 2y – 4z = –5

has infinitely many solutions. Then λ is a root of the quadratic equation.

(1) $\lambda^2 - 3\lambda - 4 = 0$ (2) $\lambda^2 - \lambda - 6 = 0$

(3) $\lambda^2 + 3\lambda - 4 = 0$ (4) $\lambda^2 + \lambda - 6 = 0$

Official Ans. by NTA (2)

6. The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2+\frac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is :

(1) 35 (2) 38

(3) 23 (4) 58

Official Ans. by NTA (2)

7. A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of the ice is 5cm, then the rate at which the thickness (in cm/min) of the ice decreases, is :

(1) $\frac{1}{9\pi}$ (2) $\frac{5}{6\pi}$

(3) $\frac{1}{18\pi}$ (4) $\frac{1}{36\pi}$

Official Ans. by NTA (3)

8. If 5x + 9 = 0 is the directrix of the hyperbola $16x^2 - 9y^2 = 144$, then its corresponding focus is :

(1) $\left(-\frac{5}{3},0\right)$ (2) (5, 0)

(3) (–5, 0) (4) $\left(\frac{5}{3},0\right)$

Official Ans. by NTA (3)

9. The sum $1+\frac{1^3+2^3}{1+2}+\frac{1^3+2^3+3^3}{1+2+3}+....$

$+\frac{1^3+2^3+3^3+....+15^3}{1+2+3+....+15}-\frac{1}{2}(1+2+3+....+15)]$

(1) 1240 (2)1860

(3) 660 (4) 620

Official Ans. by NTA (4)

10. If the line ax + y = c, touches both the curves $x^2 + y^2 = 1$ and $y^2 = 4\sqrt{2}x$, then $|c|$ is equal to :

(1) 1/2 (2) 2

(3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

Official Ans. by NTA (3)

11. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$,

where $-1 \le x \le 1, -2 \le y \le 2, x \le \frac{y}{2}$,

then for all x, y, $4x^2 - 4xy\cos\alpha + y^2$ is equal to

(1) $4\sin^2\alpha - 2x^2y^2$ (2) $4\cos^2\alpha + 2x^2y^2$

(3) $4\sin^2\alpha$ (4) $2\sin^2\alpha$

Official Ans. by NTA (3)

12. If $\int x^5e^{-x^2}dx = g(x)e^{-x^2} + c$, where c is a constant of integration, then g(–1) is equal to :

(1) $-\frac{5}{2}$ (2) 1

(3) $-\frac{1}{2}$ (4) –1

Official Ans. by NTA (1)

13. The locus of the centres of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the y-axis and lie in the first quadrant, is :

(1) $y = \sqrt{1+4x}, x \ge 0$

(2) $x = \sqrt{1+4y}, y \ge 0$

(3) $x = \sqrt{1+2y}, y \ge 0$

(4) $y = \sqrt{1+2x}, x \ge 0$

Official Ans. by NTA (4)

14. Lines are drawn parallel to the line 4x – 3y + 2 = 0, at a distance $\frac{3}{5}$ from the origin.

Then which one of the following points lies on any of these lines ?

(1) $\left(-\frac{1}{4},\frac{2}{3}\right)$ (2) $\left(\frac{1}{4},\frac{1}{3}\right)$

(3) $\left(-\frac{1}{4},-\frac{2}{3}\right)$ (4) $\left(\frac{1}{4},-\frac{1}{3}\right)$

Official Ans. by NTA (1)

15. The area (in sq. units) of the region bounded by the curves $y = 2^x$ and $y = |x + 1|$, in the first quadrant is :

(1) $\frac{3}{2}-\frac{1}{\log_e 2}$ (2) $\frac{1}{2}$

(3) $\log_e 2+\frac{3}{2}$ (4) $\frac{3}{2}$

Official Ans. by NTA (1)

16. If the plane 2x – y + 2z + 3 = 0 has the distances $\frac{1}{3}$ and $\frac{2}{3}$ units from the planes 4x – 2y + 4z + λ = 0 and 2x – y + 2z + μ = 0, respectively, then the maximum value of λ + μ is equal to :

(1) 15 (2) 5

(3) 13 (4) 9

Official Ans. by NTA (3)

17. If z and w are two complex numbers such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$, then :

(1) $\bar{z}w = i$ (2) $\bar{z}w = -i$

(3) $z\bar{w} = \frac{1-i}{\sqrt{2}}$ (4) $z\bar{w} = \frac{-1+i}{\sqrt{2}}$

Official Ans. by NTA (2)

18. Let a, b and c be in G. P. with common ratio r, where $a \neq 0$ and $0 < r \leq \frac{1}{2}$. If 3a, 7b and 15c are the first three terms of an A. P., then the 4^{th} term of this A. P. is :

(1) $\frac{7}{3}a$ (2) a

(3) $\frac{2}{3}a$ (4) 5a

Official Ans. by NTA (2)

19. The integral $\int_{\pi/6}^{\pi/3} \sec^{2/3} x \operatorname{cosec}^{4/3} x \, dx$ equal to :

(1) $3^{7/6} - 3^{5/6}$

(2) $3^{5/3} - 3^{1/3}$

(3) $3^{4/3} - 3^{1/3}$

(4) $3^{5/6} - 3^{2/3}$

Official Ans. by NTA (1)

20. Let y = y(x) be the solution of the differential equation, $\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that y(0) = 1. Then :

(1) $y'\left(\frac{\pi}{4}\right) + y'\left(\frac{-\pi}{4}\right) = -\sqrt{2}$

(2) $y'\left(\frac{\pi}{4}\right) - y'\left(\frac{-\pi}{4}\right) = \pi - \sqrt{2}$

(3) $y\left(\frac{\pi}{4}\right) - y\left(-\frac{\pi}{4}\right) = \sqrt{2}$

(4) $y\left(\frac{\pi}{4}\right) + y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{2} + 2$

Official Ans. by NTA (2)

21. Let $a_1, a_2, a_3, \ldots\ldots$ be an A. P. with $a_6 = 2$. Then the common difference of this A. P., which maximises the produce $a_1a_4a_5$, is :

(1) $\frac{6}{5}$ (2) $\frac{8}{5}$

(3) $\frac{2}{3}$ (4) $\frac{3}{2}$

Official Ans. by NTA (2)

22. The angles A, B and C of a triangle ABC are in A.P. and $a : b = 1 : \sqrt{3}$. If c = 4 cm, then the area (in sq. cm) of this triangle is :

(1) $4\sqrt{3}$ (2) $\frac{2}{\sqrt{3}}$

(3) $2\sqrt{3}$ (4) $\frac{4}{\sqrt{3}}$

Official Ans. by NTA (3)

23. Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is more than 99% is :

(1) 5 (2) 6

(3) 7 (4) 8

Official Ans. by NTA (3)

24. Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number beams is :

(1) 210 (2) 190

(3) 170 (4) 180

Official Ans. by NTA (3)

25. The sum of the real roots of the equatuion

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$$

, is equal to :

(1) 6 (2) 1

(3) 0 (4) – 4

Official Ans. by NTA (3)

26. Let $f(x) = \log_e(\sin x)$, $(0 < x < \pi)$ and $g(x) = \sin^{-1}(e^{-x})$, $(x \geq 0)$. If α is a positive real number such that $a = (fog)'(\alpha)$ and $b = (fog)(\alpha)$, then :

(1) $a\alpha^2 - b\alpha - a = 0$

(2) $a\alpha^2 + b\alpha - a = -2\alpha^2$

(3) $a\alpha^2 + b\alpha + a = 0$

(4) $a\alpha^2 - b\alpha - a = 1$

Official Ans. by NTA (4)

27. If the tangent to the curve $y = \frac{x}{x^2 - 3}$, $x \in R$, $\left(x \neq \pm\sqrt{3}\right)$, at a point $(\alpha, \beta) \neq (0, 0)$ on it is parallel to the line $2x + 6y - 11 = 0$, then :

(1) $|6\alpha + 2\beta| = 19$

(2) $|2\alpha + 6\beta| = 11$

(3) $|6\alpha + 2\beta| = 9$

(4) $|2\alpha + 6\beta| = 19$

Official Ans. by NTA (1)

28. The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is :

(1) 2 (2) 3

(3) 4 (4) 1

Official Ans. by NTA (4)

29. If $\lim\limits_{x \to 1} \frac{x^2 - ax + b}{x - 1} = 5$, then a + b is equal to :-

(1) -7 (2) -4

(3) 5 (4) 1

Official Ans. by NTA (1)

30. The negation of the boolean expression $\sim s \vee (\sim r \wedge s)$ is equivalent to :

(1) r (2) $s \wedge r$

(3) $s \vee r$ (4) $\sim s \wedge \sim r$

Official Ans. by NTA (2)