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(2:30 pm - 5:30 pm)

Question Paper

Solutions

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PHYSICS

1. The time dependence of the position of a particle of mass m=2 is given by
\[ \vec{r}(t) = 2\hat{t} - 3\hat{r} \hat{j} \]. Its angular momentum, with respect to the origin, at time t=2 is:
   1. \( 48(\hat{i} + \hat{j}) \)
   2. \( -34(\hat{k} - \hat{i}) \)
   3. \( 36\hat{k} \)
   4. \( -48\hat{k} \)

   Ans. 4
   Sol. \( \vec{L} = \vec{r} \times \vec{p} = \vec{r} \times (m \vec{v}) \)

   Where \( \vec{r} = 2t \hat{t} - 3t^2 \hat{r} \hat{j} \)

   \( \vec{v} = \frac{d\vec{r}}{dt} = 2 \hat{i} - 6t \hat{j} \)

   \( \vec{L} = m(\vec{r} \times \vec{v}) = -48\hat{k} \)

2. A bullet of mass 20g has an initial speed of \( 1 \text{ms}^{-1} \), just before it starts penetrating a mud wall of thickness 20 cm. If the wall offers a mean resistance of \( 2.5 \times 10^{-2} \text{N} \), the speed of the bullet after emerging from the other side of the wall is close to:
   1. \( 0.1 \text{ms}^{-1} \)
   2. \( 0.4 \text{ms}^{-1} \)
   3. \( 0.7 \text{ms}^{-1} \)
   4. \( 0.3 \text{ms}^{-1} \)

   Ans. 3
   Sol. \( V^2 - U^2 = 2 \times \frac{F}{m} \times s \)

   \( \Rightarrow V = 0.7 \text{ m/s} \)

3. Space between two concentric conducting spheres of radii a and b (b>a) is filled with a medium of resistivity \( \rho \). The resistance between the two spheres will be:
   1. \( \frac{\rho}{4\pi} \left( \frac{1}{a} - \frac{1}{b} \right) \)
   2. \( \frac{\rho}{2\pi} \left( \frac{1}{a} - \frac{1}{b} \right) \)
   3. \( \frac{\rho}{4\pi} \left( \frac{1}{a} - \frac{1}{b} \right) \)
   4. \( \frac{\rho}{2\pi} \left( \frac{1}{a} - \frac{1}{b} \right) \)

   Ans. 1
   Sol. \( dR = \frac{\rho \, dr}{4\pi r^2} \)

   \( R = \int_a^b dR = \int_a^b \frac{\rho \, dr}{4\pi r^2} = \frac{\rho}{4\pi} \left[ \frac{1}{a} - \frac{1}{b} \right] \)

4. A submarine experience a pressure of \( 5.05 \times 10^6 \text{Pa} \) at a depth of \( d_1 \) in a sea. When it goes further to a depth of \( d_2 \), it experiences a pressure of \( 8.08 \times 10^6 \text{Pa} \). Then \( d_2 - d_1 \) is approximately (density of water=\( 10^3 \text{kg/m}^3 \) and acceleration due to gravity=\( 10 \text{ms}^{-2} \)):
   1. 1500m
   2. 2300m
   3. 3600m
   4. 4400m

   Ans. 2
Water from a tap emerges vertically downwards with an initial speed of $1.0\text{ms}^{-1}$. The cross-sectional area of the tap is $10^{-4}m^2$. Assume that the pressure is constant throughout the stream of water and that the flow is streamlined. The cross-sectional area of the stream, 0.15m below the tap would be (Take= $g = 10\text{ms}^{-2}$)

1. $1 \times 10^{-5} m^2$
2. $2 \times 10^{-5} m^2$
3. $5 \times 10^{-5} m^2$
4. $5 \times 10^{-4} m^2$

A square loop is carrying a steady current $I$ and the magnitude of its magnetic dipole moment is $m$. If this square loop is changed to a circular loop and it carries the same current, the magnitude of the magnetic dipole moment of circular loop will be:

1. $\frac{3m}{\pi}$
2. $\frac{2m}{\pi}$
3. $\frac{4m}{\pi}$
4. $\frac{m}{\pi}$

A square loop $l \times l$ is carrying a steady current $I$ and the magnitude of its magnetic dipole moment is $m$. If this square loop is changed to a circular loop with the same current, the magnitude of the magnetic dipole moment of circular loop will be:

1. $\frac{3m}{\pi}$
2. $\frac{2m}{\pi}$
3. $\frac{4m}{\pi}$
4. $\frac{m}{\pi}$
7. Two radioactive substances A and B have decay constants $5\lambda$ and $\lambda$ respectively. At $t=0$, a sample has the same number of the two nuclei. The time taken for the ratio of the number of nuclei to become $\frac{1}{e^2}$ will be:

1. $1/4\lambda$  
2. $2/\lambda$  
3. $1/\lambda$  
4. $1/2\lambda$

Ans. 4
Sol.  
$N = N_0 e^{-\lambda t}$

\[ \frac{N_1}{N_2} = \frac{e^{-5\lambda t}}{e^{-\lambda t}} \Rightarrow \left( \frac{1}{e} \right)^2 = e^{-4\lambda t} \]

$\Rightarrow t = \frac{1}{2\lambda}$

8. In an experiment, brass and steal wires of length 1 m each with areas of cross section 1 mm$^2$ are used. The wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to elongation. The stress required to produce a net elongation of 0.2 mm is: {Given the young’s Modulus for steel and brass are, respectively $120 \times 10^9 N/m^2$ and $60 \times 10^9 N/m^2$} 

1. $1.2 \times 10^6 N/m^2$  
2. $0.2 \times 10^6 N/m^2$  
3. $1.8 \times 10^6 N/m^2$  
4. $4.0 \times 10^6 N/m^2$

Ans. 4  
Sol.  
$e = e_1 + e_2$

\[ e = \frac{F}{A} \left[ \frac{l}{Y_1} + \frac{l}{Y_2} \right] \]

$\Rightarrow 0.2 \times 10^{-3} = \left( \text{stresses} \right)(1) = \left[ \frac{l}{Y_1} + \frac{l}{Y_2} \right]$  

$\Rightarrow \text{Stress} = 4.0 \times 10^6 N/m^2$

9. The graph shows how the magnification $m$ produced by a thin lens varies with image distance $v$. What is the focal length of the lens used?

![Graph showing magnification m vs. image distance v]

1. $\frac{b^2 c}{a}$  
2. $\frac{a}{c}$  
3. $\frac{b^2}{ac}$  
4. $\frac{b}{c}$

Ans. 4
Sol. \[ m = \frac{f - V}{f} = 1 - \frac{1}{f} \]

\[ m = 1 - \frac{1}{f} \]

\[ \therefore \text{slope} = \frac{c}{b} = -\frac{1}{f} \]

\[ \therefore f = -\frac{b}{c} \]

10. The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod if it is to support a 400 N Load without exceeding its elastic limit?

1. 1.36mm
2. 2.116mm
3. 3.090mm
4. 4.100mm

Ans. 2

Sol. Elastic unit = \( \frac{F}{A} \)

\[ 379 \times 10^6 = \frac{400}{\pi r^2} \]

\[ r = 0.57 \times 10^{-3} \text{ m} \]

Diameter = \( 2 \times 0.57 \times 10^{-3} \text{ m} \)

= 1.16 mm

11. A coil of self inductance 10 mH and resistance 0.1 \( \Omega \) is connected through a switch to a battery of internal resistance 0.9 \( \Omega \). After the switch is closed, the time taken for the current to attain 80% of the saturation value is: \( \{ \text{take ln5} = 1.6 \} \)

1. 0.324s
2. 0.016s
3. 0.002s
4. 0.103s

Ans. 2

Sol.

\[ L \frac{di}{dt} + i (R + r) = \varepsilon \]

\[ \Rightarrow i = \frac{\varepsilon}{R + r} \left( 1 - e^{-\frac{R + r}{L}} \right) \]

Let ‘t’ is the time taken to get 80% of saturation current

\[ \Rightarrow \frac{80}{100} i_o = i_o \left( 1 - e^{-\frac{1}{10^7}} \right) \Rightarrow 0.8 = 1 - e^{-10^7} i_o \]

\[ \Rightarrow 10^7 i_o = \ln(5) \Rightarrow i_o = 1.6 \times 10^{-7} \text{ sec} = 0.016 \text{ sec} \]

12. In Li⁺⁺ electron in first Bohr orbit is excited to a level by a radiation of wavelength \( \lambda \) when the ion gets deexcited to the ground state in all possible ways (including intermediate emissions) a total of six spectral lines are observed. What is the value of \( \lambda \) ? \( \{ \text{Given} \ h = 6.63 \times 10^{-34} \text{ Js}; c = 3 \times 10^8 \text{ m/s} \} \)

1. 1.108nm
2. 2.94nm
3. 3.114nm
4. 4.123nm

Ans. 1
Sol. \( E_n = -\frac{Z^2}{n^2} (13.6 \text{ ev}) \Rightarrow \frac{1}{\lambda} = Z^2 \left( \frac{13.6 \text{ eV}}{\hbar c} \right) \left( \frac{1}{n^2} - \frac{1}{n_1^2} \right) \)

\[ \Rightarrow \lambda = \frac{914A^0}{Z^2} \left( \frac{1}{n^2} - \frac{1}{n_1^2} \right) \]

And given \( ^* \mathbf{C}_2 = 6 \Rightarrow n_2 = 4 \)

\[ \Rightarrow \lambda = \frac{914}{9} \left( \frac{1}{16} - \frac{1}{1} \right) \times \frac{16}{15} = 108.3 \text{ Å} = 10.8 \text{ nm} \]

13. In the formula \( X = 5YZ^2 \), \( X \) and \( Y \) have dimensions of capacitance and magnetic field, respectively. What are the dimensions of \( Y \) in SI units?

1. \( [M^{-2}L^2T^6A^3] \)  
2. \( [M^{-2}L^4T^{-2}A^{-2}] \)  
3. \( [M^{-1}L^2T^4A^{-3}] \)  
4. \( [M^{-3}L^2T^8A^4] \)

Ans. 4

Sol. Given formula \( X = 5YZ^2 \)

\[ \Rightarrow [Y] = [XZ^{-2}] \]

\[ \Rightarrow [Y] = \left[ \frac{M^{-1}L^2T^4A^2}{M^2L^2T^4A^2} \right] \]

\[ \Rightarrow [Y] = [M^{-3}L^{-2}T^8A^4] \]

14. A cubic block of side 0.5m floats on water with 30\% of its volume under water. What is the maximum weight that can be put on the block without fully submerging it under water? (Take, density of water = 10\(^3\) kg/m\(^3\))

1. 1.875kg  
2. 2.654kg  
3. 3.301kg  
4. 4.463kg

Ans. 1

Sol. Extra weight can be put on the block = 70 \% \( V_{\text{block}} \rho_{\text{water}} \times g \)

\[ = \frac{7}{100} \times 0.125 \times 1000 \times g = 12.5 \times 7 \times g = 87.5 \times g \]

= 87.5 KgwT

15. Light is incident normally on a completely absorbing surface with an energy flux of 25\( \text{W cm}^{-2} \). If the surface has an area of 25\( \text{cm}^2 \), the momentum transferred to the surface in 40 min time duration will be

1. 6.3 \times 10^{-4} \text{Ns}  
2. 3.5 \times 10^{-6} \text{Ns}  
3. 5.0 \times 10^{-3} \text{Ns}  
4. 1.4 \times 10^{-6} \text{Ns}

Ans. 3

Sol. Given intensity of light \( I = 25 \text{W cm}^{-2} = 25 \times 10^4 \text{ W/m}^2 \)

\[ A = 25 \text{ cm}^2 = 25 \times 10^{-4} \text{ m}^2 \]

\( \Rightarrow \) including energy rate = 625 \times 10^3 \text{ W} \)

\( \Rightarrow \) Momentum transferred = \( \frac{E}{C} = \frac{625 \times 40 \times 60}{3 \times 10^9} \)

= 5 \times 10^{-3} \text{ Ns}
16. In free space, a particle A of charge $1\mu C$ is held fixed at a point P. Another particle B of the same charge and mass $4\mu g$ is kept at a distance of 1 mm from P. If B is released, then its velocity at a distance of 9 mm from P is: {Take $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 Nm^2C^{-2}$}

1. $2.0 \times 10^3 m/s$
2. $3.0 \times 10^4 m/s$
3. $1.0 m/s$
4. $1.5 \times 10^2 m/s$

Ans. 1

Sol. Applying conservation of energy

$$\frac{KQq}{r_1} + 0 + 0 = \frac{KQq}{r_2} + 0 + \frac{1}{2}mv^2$$

$$\Rightarrow \frac{9 \times 10^9 \times 1 \times 10^{-12}}{1 \times 10^{-3}} = \frac{9 \times 10^9 \times 1 \times 10^{-12}}{9 \times 10^{-3}} + \frac{1}{2} \times 4 \times 10^{-6} \times v^2$$

$$\Rightarrow 2 \times 10^{-6}v^2 = 8 \Rightarrow v = 2 \times 10^3 m/s$$

17. A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number of complete revolutions made by the spaceship in 24 hours around the planet? {Given= Mass of planet= $8 \times 10^{22} kg$, radius of planet= $2 \times 10^6 m$, Gravitational constant $G = 6.67 \times 10^{-11} Nm^2/kg^2$}

1. $1.13$
2. $2.17$
3. $3.11$
4. $4.9$

Ans. 3

Sol. As

$$T = 2\pi \sqrt{\frac{r^3}{Gm}} \Rightarrow T = 2\pi \times \sqrt{\frac{(2 \times 10^6)^3}{6.67 \times 10^{-11} \times 8 \times 10^{22}}}$$

$$T = \frac{2 \pi \times 2871 \times 10^6}{23 \times 10^8} = 7840 sec$$

$$\Rightarrow \text{No. of revolutions in 24 hrs} = \frac{24 \times 60 \times 60}{7840} = 11.02$$

18. When heat Q is supplied to a diatomic gas of rigid molecules, at constant volume its temperature increases by $\Delta T$. The heat required to produce the same change in temperature, at a constant pressure is:

1. $\frac{3}{2} Q$
2. $\frac{7}{5} Q$
3. $\frac{5}{3} Q$
4. $\frac{2}{3} Q$

Ans. 2

Sol. Given $Q = nC_v \Delta T \Rightarrow Q' = nC_p \Delta T = ?$

$$\Rightarrow Q' = \frac{C_p}{C_v} Q = \frac{7}{5} Q$$
19. A solid sphere of mass $M$ and radius $R$ is divided into two unequal parts. The first part has a mass of $\frac{7M}{8}$ and is converted into a uniform disc of radius $2R$. The second part is converted into a uniform solid sphere. Let $I_1$ be the moment of inertia of the disc about its axis and $I_2$ be the moment of inertia of the new sphere about its axis. The ratio $I_1/I_2$ is given by:

\[
\frac{I_1}{I_2} = \frac{1}{2} \left( \frac{m_1 r_1^2}{2} \right) = \frac{1}{2} \left( \frac{7M}{8} \right) \left( \frac{2R}{2} \right)^2
\]

\[
= \frac{1}{2} \left( \frac{7}{8} \right) \left( \frac{4}{1} \right) = 3.57
\]

An. 4

Sol. $I_1 = \frac{1}{2}m_1r_1^2$  
$I_2 = \frac{2}{5}m_2r_2^2$

$\Rightarrow \frac{I_1}{I_2} = \frac{\frac{1}{2} \cdot \frac{7m}{8} \cdot (2R)^2}{\frac{2}{5} \cdot \frac{m}{2} \cdot R^2} = \frac{1}{2} \times \frac{7}{8} \times 4 \times \frac{8}{2} \times \frac{4}{1} = 140$

20. A source of sound $S$ is moving with a velocity of 50 m/s towards a stationary observer. The observer measures the frequency of the source as 1000 Hz. What will be the apparent frequency of the source when it is moving away from the observer after crossing him? (take velocity of sound in air is 350 m/s)

\[
f_1 = 1000 \Rightarrow \frac{350}{350-50} f_1 = \frac{350}{350+50} f_2 = ?
\]

\[
f_2 = \frac{350-50}{350+50} \times 1000 = \frac{300}{400} \times 1000 = 750 \text{ Hz}
\]

An. 3

Sol. Given $\frac{350}{350-50} f_1 = 1000 \Rightarrow \frac{350}{350+50} f_2 = ?$

21. One mole of an ideal gas passes through a process where pressure and volume obey the relation $P = P_0 \left[ 1 - \frac{1}{2} \left( \frac{V_0}{V} \right)^2 \right]$ $I_1/I_2$. Here $P_0$ and $V_0$ are constants, calculate the change in the temperature of the gas if its volume changes from $V_0$ to $2V_0$

1. $\frac{P_0 V_0}{4} / R$  
2. $\frac{5 P_0 V_0}{4} / R$  
3. $\frac{1 P_0 V_0}{2} / R$  
4. $\frac{3 P_0 V_0}{4} / R$

An. 2

Sol. Given $P = P_0 \left[ 1 - \frac{1}{2} \left( \frac{V_0}{V} \right)^2 \right]$

As $PV = nRT$ ( $n = 1$ mole )

\[
P = \frac{RT}{V}
\]

\[
\frac{RT}{V} = P_0 - \frac{P_0 V_0^2}{2 V^2}
\]
\[ T = \frac{1}{R} \left[ P_0 V - \frac{P_0 V_0^2}{2V} \right] \]

\[ V = V_0 \]

\[ T_1 = \frac{1}{R} \left[ \frac{P_0 V_0}{2} \right] \]

\[ V = 2V_0 \]

\[ T_2 = \frac{1}{R} \left[ 2P_0 V_0 - \frac{P_0 V_0}{4} \right] \]

\[ T_2 = \frac{7P_0 V_0}{4R} \]

\[ \Delta T = T_1 \sim T_2 = \frac{5P_0 V_0}{4R} \]

22. A plane is inclined at an angle \( \alpha = 30^\circ \) with respect to the horizontal. A particle is projected with a speed \( u = 2 \text{m/s} \) from the base of the plane, making an angle \( \theta = 15^\circ \) with respect to the plane as shown in the figure. The distance from the base, at which the particle hits the plane is close to (take \( g = 10 \text{ms}^{-2} \})

\[
\begin{align*}
R &= \frac{2u^2 \sin \alpha \cos(\alpha + \beta)}{g \cos^3 \beta} \\
\text{Here} & \quad \alpha = \theta = 15^\circ \\
\beta &= \alpha = 30^\circ \\
R &= \frac{2 \times 4 \times \sin 15^\circ \cos 45^\circ}{g \cos^3 30^\circ} \\
&= 20 \text{ cm}
\end{align*}
\]

23. A 2mW laser operates at a wavelength of 500nm. The number of photons that will be emitted per second is: (Given plank’s constant \( h = 6.6 \times 10^{-34} \text{Js} \), speed of light \( c = 3.0 \times 10^8 \text{m/s} \))
1. \(5 \times 10^{15}\)  
2. \(1 \times 10^{16}\)  
3. \(2 \times 10^{16}\)  
4. \(1.5 \times 10^{16}\)

Ans. 1

Sol. Power = \(\frac{nE}{t}\)

\[
\frac{n}{t} \cdot \frac{h}{\nu} = \frac{n}{t} \cdot \frac{h}{\lambda}
\]

\[
\left(\frac{n}{t}\right) = \frac{P \lambda}{hc} = \frac{2 \times 10^{-3} \times 500 \times 10^{-9} \cdot m}{6.6 \times 10^{-34} \times 3 \times 10^{8}}
\]

\[
= 5 \times 10^{15}
\]

24. Two blocks A and B of mass \(m_A = 1\) kg and \(m_B = 3\) kg are kept on the table as shown in figure. The coefficient of friction between A and B is 0.2 and between B and the surface of the table is also 0.2. The maximum force \(F\) that can be applied on B horizontally, so that the block A does not slide over the block B is \(\text{Take} \ g = 10m/s^2\)

<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1.8N</td>
</tr>
<tr>
<td>2.</td>
<td>2.40N</td>
</tr>
<tr>
<td>3.</td>
<td>3.12N</td>
</tr>
<tr>
<td>4.</td>
<td>4.16N</td>
</tr>
</tbody>
</table>

Ans. 4

Sol.

\[
M_A = 1 \text{ kg} \]

\[
M_B = 3 \text{ kg} \]

Pseudo force on A

\[
M_A a_{\text{system}} = \mu m_A g
\]

\[
a = 0.2 \times 1 \times 10
\]

\[
a = 2 \text{ m/sec}^2
\]

Taking (A+B) as system

\[
F - \mu . (m_A + m_B) g = (m_A + m_B) a_{\text{system}}
\]

\[
F - (0.2)(4)(10) = 4(2)
\]
25. The magnitude of the magnetic field at the centre of an equilateral triangular loop of side 1 m which is carrying a current of 10A is: \( \mu_0 = 4\pi \times 10^{-7} \text{N} \cdot \text{A}^{-2} \) \( B = \frac{\mu_0 I}{4\pi \cdot \frac{1}{2\sqrt{3}}} \) \( = 18 \times 10^{-6} \text{ Tesla} \) \( = 18 \mu T \) 

Ans. 1

Sol.

\[ B^1 = 3B \]
\[ = 3 \times \frac{\mu_0 I}{4\pi \cdot \frac{1}{2\sqrt{3}}} \left( \sin 60^\circ + \sin 60^\circ \right) \]
\[ = 18 \times 10^{-6} \text{ Tesla} \]
\[ = 18 \mu T \]

26. The figure represents a voltage regulator circuit using a Zener diode. The breakdown voltage of the zener diode is 6V and the load resistance is \( R_L = 4k\Omega \). The series resistance of the circuit is \( R_i = 1k\Omega \). If the battery voltage \( V_B \) varies from 8V to 16V, what are the minimum and maximum values of the current through zener diode?

\[
\text{Options:} \\
1. 0.5mA; 8.5mA \\
2. 1.5mA; 8.5mA \\
3. 0.5mA; 6mA \\
4. 1mA; 8.5mA
\]

Ans. 1
2019 Jee-Main

**Sol.**

If \( V = 6V \quad I_L = 1.5 mA \)
\( I_Z = 0.5 mA \quad I = 2 mA \)

If \( V = 16V \quad I_L = 1.5 mA \)
\( I_Z = 8.5 mA \quad I = 10 mA \)

27. A metal of mass 5 g and radius 1 cm is fixed to a thin stick AB if negligible mass as shown in the figure. The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5s, is close to:

1. \( 4.0 \times 10^{-6} Nm \)  
2. \( 2.0 \times 10^{-5} Nm \)  
3. \( 1.6 \times 10^{-5} Nm \)  
4. \( 7.9 \times 10^{-6} Nm \)

**Ans.** 2

**Sol.**

\( w = w_0 + \alpha t \)
\( (25)(2\pi) = 0 + \alpha (5) \)
\( \alpha = 10\pi \text{ rad/sec}^2 \)

\( \tau = I\alpha \)
\( = \left( \frac{MR^2}{4} + MR^2 \right)(10\pi) \)
\( = \left( \frac{5}{4} MR^2 \right)(10\pi) \)
\( = \frac{5}{4} \times 5 \times 10^{-3} \times \left(1 \times 10^{-2}\right)^2 \times 10 \times \pi \)
\( = 2 \times 10^{-5} Nm \)

28. A simple pendulum of length L is placed between the plates of a parallel plate capacitor having electric field E, as shown in figure. Its bob has mass m and change q, The time period of the pendulum is given by:
1. $2\pi \sqrt{\frac{L}{g^2 + \left(\frac{qE}{m}\right)^2}}$

2. $2\pi \sqrt{\frac{L}{g + \frac{qE}{m}}}$

3. $2\pi \sqrt{\frac{L}{g - \frac{qE}{m}}}$

4. $2\pi \sqrt{\frac{L}{g^2 - \frac{q^2E^2}{m^2}}}$

Ans. 1

Sol. $a = \sqrt{g^2 + \left(\frac{qE}{m}\right)^2}$

$T = 2\pi \sqrt{\frac{L}{g^2 + \left(\frac{qE}{m}\right)^2}}$

29. In a Young’s double slit experiment, the ratio of the slit’s width is 4:1. The ratio of the intensity of maxima to minima, close to the central fringe on the screen, will be:

1. $\left(\sqrt{3} + 1\right)^4 : 16$
2. 2.25:9
3. 3.9:1
4. 4.4:1

Ans. 3

Sol. $I_{\text{max}} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$I_{\text{min}} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

If $I \propto$ slit width

Then ratio $= \left(\frac{2 + 1}{2 - 1}\right)^2 = 9:1$

$I \propto A^2 \propto (\text{slit width})^2$

Ratio $= \left(\frac{4 + 1}{4 - 1}\right)^2 = \frac{25}{9}$

Ratio = 25 : 9
2 (or) 3

As of now according to Jee key option (2) correct

30. The correct figure that shows, schematically, the wave pattern produced by superposition of two waves of frequencies 9 Hz and 11Hz is

1.  
2.  
3.  
4.  

Ans.

Sol. \( f_b = 2\text{Hz} \)

Two maximas in one second 2 option

CHEMISTRY

31. The major product obtained in the given reaction is

\[
\text{CH}_3\text{O} - \text{CH}_2 - \text{CH}_2 - \text{CH}_3 + \text{AlCl}_3 \rightarrow \text{Product}
\]

1.  
2.  
3.  
4.  

Ans. 1
Sol. Friedal craft’s alkylation

\[ \text{Orient. is decided by SAG.} \]

32. The major product ‘Y’ in the following reaction is:

\[ \text{Ans. 1} \]

Sol. With NaOCl halo form reaction

\[ \text{Ans. 1} \]
33. Which of these factors does not govern the stability of a conformation in acyclic compounds
   1. Torsional strain  
   2. Steric interactions  
   3. Electrostatic forces of interactions  
   4. Angle strain  
   Ans. 4  
   Sol. Angle strain is absent in acyclic compounds.

34. Air pollution that occurs in sunlight is:
   1. Oxidising smog  
   2. Reducing smog  
   3. fog  
   4. Acid rain  
   Ans. 1  
   Sol. Photochemical smog occurs in warm, dry and sunny climate. It has high concentration of oxidizing agents and is called as oxidizing smog.

35. The correct option among the following is:
   1. Colloidal particles in lyophobic sols can be precipitated by electrophoresis.
   2. Brownian motion in colloidal solution is faster if the viscosity of the solution is very high.
   3. Addition of alum water makes it unfit for drinking.
   4. Colloidal medicines are more effective because they have small surface area.
   Ans. 1  
   Sol. Lyophobic colloids may migrate in either direction (or) may not migrate at all whereas lyophobic particles migrate in only one direction in electric field.

36. The noble gas that does NOT occur in the atmosphere is:
   1. Ne  
   2. He  
   3. Kr  
   4. Ra  
   Ans. 1  
   Sol. Radioactive noble gas radon is not available in air.

37. Which of the following graphs between molar conductivity \( (\Lambda_m) \) versus \( \sqrt{C} \) is correct?
38. In chromatography, which of the following statements is INCORRECT for $R_f$?

1. $R_f$ value depends on the type of chromatography
2. $R_f$ value is dependent on the mobile phase
3. Higher $R_f$ value means higher adsorption
4. The value of $R_f$ can not be more than one

Ans. 3

Sol. distance moved by the substance from baseline distance moved by solved from baseline It adsorption ability in less then RF is higher

39. A hydrated solid X on heating initially gives a monohydrated compound X,Y upon heating above 373 K leads to an anhydrous white powder Z, X and Z respectively are:

1. Baking soda and dead burnt plaster.
2. Baking soda and soda ash
3. Washing soda and dead burnt plaster
4. Washing soda and soda ash

Ans. 4
40. Points I, II and III in the following plot respectively correspond to 
\( V_{mp} \text{: most probable velocity} \)

1. \( V_{mp} \text{ of } N_2 (300K) ; V_{mp} \text{ of } O_2 (400K) ; V_{mp} \text{ of } H_2 (300K) \)
2. \( V_{mp} \text{ of } N_2 (300K) ; V_{mp} \text{ of } H_2 (300K) ; V_{mp} \text{ of } O_2 (400K) \)
3. \( V_{mp} \text{ of } H_2 (300K) ; V_{mp} \text{ of } N_2 (300K) ; V_{mp} \text{ of } O_2 (400K) \)
4. \( V_{mp} \text{ of } O_2 (400K) ; V_{mp} \text{ of } N_2 (300K) ; V_{mp} \text{ of } H_2 (300K) \)

Ans. 1

Sol. \( H_2 = \sqrt{\frac{300}{2}} = \sqrt{150} \quad MPV = \sqrt{\frac{2RT}{M}} \)
\[ O_2 = \sqrt{\frac{400}{32}} = \sqrt{\frac{100}{8}} = \sqrt{12.5} \]
\[ N_2 = \sqrt{\frac{300}{28}} = \sqrt{\frac{75}{7}} = \sqrt{10.7} \]

41. Number of stereo centers present in linear and cyclic structures of glucose are respectively
1. 5 & 5  
2. 4 & 4  
3. 5 & 4  
4. 4 & 5

Ans. 4
42. The minimum amount of $O_2(g)$ consumed per gram of reactant is for the reaction:
(Given atomic mass : Fe = 56, O=16, Mg=24, P= 31, C=12, H=1)

1. $2 \text{Mg}(s) + O_2(g) \rightarrow 2 \text{MgO}(s)$
2. $4 \text{Fe}(s) + 3 O_2(g) \rightarrow 2 \text{Fe}_2O_3(s)$
3. $P_4(s) + 5 O_2(g) \rightarrow P_4O_{10}(s)$
4. $C_3H_8(g) + 5 O_2(g) \rightarrow 3 \text{CO}_2(g) + 4 \text{H}_2O(l)$

Ans. 2

Sol. (i) $1 \text{gm of Mg} \rightarrow \frac{2}{3} \text{gm of O}_2$
(ii) $1 \text{gm of Fe} \rightarrow \frac{3}{7} \text{gm of O}_2$
(iii) $1 \text{gm of P} \rightarrow \frac{160}{124} \text{gm of O}_2$
(iv) $1 \text{gm of C}_3H_8 \rightarrow \frac{160}{44} \text{gm of O}_2$

43. The INCORRECT statement is:
1. The gemstone, ruby has $Cr^{3+}$ ions occupying the octahedral sites of beryl
2. The spin – only magnetic moments of $[Fe(H_2O)_6]^{2+}$ and $[Cr(H_2O)_6]^{2+}$ are nearly similar
3. The color of $[CoCl(NH_3)_5]^{2+}$ is violet as it absorbs the yellow light
4. The spin only – magnetic moment of $[Ni(NH_3)_4(H_2O)_2]^{2+}$ is 2.83 BM.

Ans. 1
Sol.  
1) ruby – alumina with chromium  
2) each complex - 4 unpaired \( \bar{\epsilon} \) in each  
3) The color of \( [CoCl(NH_3)_5]^{2+} \) is violet as it absorbs the yellow light  
4) outer orbital complex – 2 unpaired \( \bar{\epsilon} \)  

44. The crystal field stabilization energy (CFSE) of \( [Fe(H_2O)_6]Cl_2 \) and \( K_2[NICl_4] \), respectively are:

1. \(-2.4\Delta_o \) and \(-1.2\Delta_t \)  
2. \(-0.4\Delta_o \) and \(-1.2\Delta_t \)  
3. \(-0.6\Delta_o \) and \(-0.8\Delta_t \)  
4. \(-0.4\Delta_o \) and \(-0.8\Delta_t \)  

Ans. 4  

Sol.  
\[ \Delta_o = d^6 \ t_{2g}^4 \ v_e^2 \ : \ [(-0.4) + 2(0.6)] \Delta_o \]  
\[ \Delta_t = d^8 \ e_g^4 \ t_{2g}^4 \ : \ [(-0.6) + 0.4) + 4] \Delta_t = -0.8\Delta_t \]  

45. The increasing order of nucleophilicity of the following nucleophiles is:  
a) \( CH_3CO_2^- \)  
b) \( H_2O \)  
c) \( CH_3SC\) \( \bar{\bar{\sigma}}\)  
d) \( OH^{-} \)  

1. \( b < c < a < d \)  
2. \( a < d < c < b \)  
3. \( d < a < c < b \)  
4. \( b < c < d < a \)  

Ans. 1  

Sol.  
\[ H_2O < CH_3SO_3^- < CH_3CO_2^- < OH^- \]  

46. The pH of a 0.02 M \( NH_4Cl \) solution will be  
[given \( K_b(NH_4OH) = 10^{-5} \) and \( \log 2 = 0.301 \)]  

1. 5.35  
2. 2.65  
3. 4.65  
4. 4.35  

Ans. 1  

Sol.  
\[ [H^+] = \sqrt{\frac{K_w \cdot C}{K_b}} \]  

47. 1 g of a non-volatile non – electrolyte solute is dissolved in 100 g of two different solvents A and B whose ebullioscopic constants are in the ratio of 1:5.  
The ratio of the elevation in their boiling points, \( \frac{\Delta T_b(A)}{\Delta T_b(B)} \), is:  

1. 1: 0.2  
2. 10:1  
3. 5:1  
4. 1:5  

Ans. 4
48. The correct order of the first ionization enthalpies is:
1. Ti < Mn < Ni < Zn  
2. Mn < Ti < Zn < Ni  
3. Ti < Mn < Zn < Ni  
4. Zn < Ni < Mn < Ti
Ans. 1

Sol. \[
\frac{\Delta T_b(A)}{\Delta T_b(B)} = \frac{K_p(A)}{K_p(B)} = \frac{1}{5}
\]

49. The major product ‘Y’ in the following reaction is:

\[
\begin{align*}
\text{Cl} & \quad \text{EtONa} & \quad \text{Heat} & \quad \text{HBr} & \quad \text{Y} \\
\text{Br} & \quad & & & \\
\text{HO} & \quad & & & \\
\end{align*}
\]

1.  
2.  
3.  
4.  
Ans. 2

Sol. 

\[
\begin{align*}
\text{CH}_3 & \quad \text{CH}_3 & \quad \text{C}_2\text{H}_5 \quad \text{O} + \\
\text{Cl} & & & \\
\text{CH} & \quad \text{CH} & \quad \text{CH} & \quad \text{CH}_3 \\
\end{align*}
\]
50. For the reaction \( 2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g) \), \( \Delta H = -57.2 \text{kJ mol}^{-1} \) and 
\( K_c = 1.7 \times 10^6 \). Which of the following statement is INCORRECT?
1. The equilibrium constant is large suggestive of reaction going to completion and so no catalyst is required.
2. The equilibrium constant decreases as the temperature increases.
3. The equilibrium will shift in forward direction as the pressure increases.
4. Volume will not affect the equilibrium constant.

Ans. 1

Sol. \( K_c \) and \( \Delta H \) are not related to kinetic factor.

51. The number of pentagons in \( C_{60} \) and trigons (triangles) in white phosphours, respectively, are:
1. 20 and 4  
2. 20 and 3  
3. 12 and 4  
4. 12 and 3

Ans. 3

Sol. No. of pentagon in \( C_{60} = 12 \)
No. of trigon in white \( P_4 = 4 \)

52. The correct statements among (a) to (d) are:
   a) Saline hydrides produce \( H_2 \) gas when reacted with \( H_2O \)
   b) Reaction of \( \text{LiAlH}_4 \) with \( \text{BF}_3 \) leads to \( \text{B}_2\text{H}_6 \)
   c) \( \text{PH}_3 \) and \( \text{CH}_4 \) are electron–rich and electron–precise hydrides, respectively.
   d) \( \text{HF} \) and \( \text{CH}_4 \) are called as molecular hydrides.

1. a, b, c and d  
2. a, b and c only  
3. a, c and d only  
4. c and d only

Ans. 1

Sol. 1) Saline hydrides, alkali and alkanine metal hydride
\( \text{NaH} + H_2O \rightarrow \text{NaOH} + H_2 \)
\( \text{CaH}_2 + H_2O \rightarrow \text{Ca(OH)}_2 + H_2 \)
2) \( 4\text{BF}_3 + 3\text{LiAlH}_4 \rightarrow 3\text{LiF} + 2\text{B}_2\text{H}_6 + 3\text{AlF}_3 \)
3) \( \text{PH}_3 \) electron rich hydrides - \( \text{CH}_4 \) electron precise hydride -

\[
\begin{align*}
\text{H} & \quad \text{P} \quad \text{H} \\
\text{H} & \quad \text{C} \quad \text{H} \\
\text{H} & \quad \text{H}
\end{align*}
\]

4) HF and CH\(_4\) are called as molecular hydride

53. For the reaction of \( \text{H}_2 \) with \( \text{I}_2 \), the rate constant is \( 2.5 \times 10^{-4} \text{ dm}^3 \text{mol}^{-1} \text{s}^{-1} \) at 327\(^\circ\)C and \( 1.0 \text{ dm}^3 \text{mol}^{-1} \text{s}^{-1} \) at 527\(^\circ\)C. The activation energy for the reaction, in \( \text{kJ mol}^{-1} \) is:

- 1. 166
- 2. 150
- 3. 59
- 4. 72

Ans. 1

Sol.

\[
\log \frac{k_2}{k_1} = \frac{E_a}{2.303[R]} \left[ \frac{1}{T_1} - \frac{1}{T_2} \right]
\]

\[
\log \frac{1}{2.5 \times 10^{-4}} = \frac{E_a}{2.303[8.314]} \left[ \frac{1}{600} - \frac{1}{800} \right]
\]

\[E_a = 166 \text{ KJ} \text{ mol}^{-1}\]

54. The ratio of the shortest wavelength of two spectral series of hydrogen spectrum is found to be about 9. The spectral series are

- 1. Paschen and Pfund
- 2. Lyman and Paschen
- 3. Brackett and Pfund
- 4. Balmer and Brackett

Ans. 2

Sol. Shortest wavelength of lymen series

\[
\frac{1}{\lambda_1} = R_H \left[ \frac{1}{1^2} - \frac{1}{2} \right]
\]

\[
\frac{1}{\lambda_1} = R_l \Rightarrow \lambda_1 = \frac{1}{R_H}
\]

Shortest wavelength of pascham series

\[
\frac{1}{\lambda_2} = R_H \left[ \frac{1}{9} - \frac{1}{\infty^2} \right] \Rightarrow \frac{1}{\lambda_2^2} = \frac{R_H}{9}
\]

\[
\frac{1}{\lambda_2} = \frac{R_H}{9} \Rightarrow \lambda_2 = \frac{9}{R_H}
\]
55. Compound \( A(C_9H_{10}O) \) shows positive iodoform rest. Oxidation of \( A \) with \( KMnO_4 / KOH \) gives acid \( B(C_6H_6O_4) \). Anhydride of \( B \) is used for the preparation of phenolphthalein. Compound \( A \) is:

1. \[ \text{CH}_2 = \text{C}(\text{H}) \]
2. \[ \text{O} \]
3. \[ \text{CH}_3 \]
4. \[ \text{O} \]

Ans. 4

Sol.

\[ \text{C}_3\text{H}_3\text{O} \text{NaIO} \text{H}_2\text{O} \text{NaBH}_4 \text{P}_4\text{O}_{10} \]

56. Which of the following is NOT a correct method if the preparation of benzylamine from cyanobenzene?

1. \( H_2 / Ni \)
2. (i) \( \text{LiAlH}_4 \) (ii) \( H_3O^+ \)
3. (i) \( \text{SnCl}_2 + \text{HCl} \text{ (gas)} \) (ii) \( \text{NaBH}_4 \)
4. (i) \( \text{HCl} / H_2O \) (ii) \( \text{NaBH}_4 \)
57. The correct statement is:
1. Zincite is a carbonate ore.
2. Zone refining process is used for the refining of titanium.
3. Aniline is a froth stabilizer.
4. Sodium cyanide cannot be used in the metallurgy of silver.

Ans. 3

Sol. Aniline is a froth stabilizer.

58. The correct match between item – I and Item – II is

<table>
<thead>
<tr>
<th>Item-I</th>
<th>Item-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) High density polythene</td>
<td>I) Peroxide catalyst</td>
</tr>
<tr>
<td>b) Polyacrylonitrile</td>
<td>II) Condensation at high temperature &amp; pressure</td>
</tr>
<tr>
<td>c) Novolac</td>
<td>III) Ziegler–Natta Catalyst</td>
</tr>
<tr>
<td>d) Nylon 6</td>
<td>IV) Acid or base catalyst</td>
</tr>
<tr>
<td>1. a-III, b-I, c-II, d-IV</td>
<td>2. a-II, b-IV, c-II, d-IV</td>
</tr>
<tr>
<td>3. a-IV, b-IV, C-I, d-III</td>
<td>4. a-III, b-I, c-IV, d-II</td>
</tr>
</tbody>
</table>
Ans. 4
Sol. High density polythene is obtained by using zegler natta catalyst
Polyacrolol nitrile is obtained by acrylonitrile using peroxide as catalyst.
Navalac is obtained by the $H^+(or)OH^-$

\[
\text{Catalysed polymerization of } \text{OH} \quad \text{and} \quad \text{H} \quad \text{H} \\
\text{Nylon-6 is condensation polymerization of caprolactum at high tem and pressure}
\]

59. The highest possible oxidation states of uranium plutonium, respectively are
1. 6 and 7  
2. 7 and 6  
3. 4 and 6  
4. 6 and 4

Ans. 1
Sol. Highest Oxidation state of uranium = +6
Oxidation stat of plutonium = +7

60. The difference between $\Delta H$ and $\Delta U$ ($\Delta H - \Delta U$), when the combustion of one mole heptane (I) is carried out at a temperature T, is equal to:
1. -3RT  
2. -4RT  
3. 3RT  
4. 4R

Ans. 2
Sol. 
\[
C_7H_{16(i)} + 11O_2 \rightarrow 7CO_2 + 8H_2O \rightarrow \\
\Delta n_{\text{gas}} = 7 - 11 \\
\Delta H = \Delta U + \Delta nRT \\
\Delta H - \Delta V = -4RT
\]

MATHEMATICS

61. The sum of the real roots of the equation 
\[
\begin{vmatrix}
 x & -6 & -1 \\
 2 & -3x & x-3 \\
 -3 & 2x & x+2 \\
\end{vmatrix} = 0 \text{, is equal to}
\]
1. 0  
2. 6  
3. -4  
4. 1

Ans. 1
Sol. 
\[
x(-3x^2 - 6x - 2x^2 + 6x) + 6(2x + 4 + 3x - 9) - 1(4x - 9x) = 0 \\
\text{Coefficient of } x^2 = 0
\]
62. A perpendicular is drawn from a point on a line \( \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{1} \) to the plane \( x + y + z = 3 \) such that the foot of the perpendicular Q also lies on the plane \( x - y + z = 3 \).

Then the coordinates of Q are:

1. \((4,0,-1)\)
2. \((2,0,1)\)
3. \((1,0,2)\)
4. \((-1,0,4)\)

Ans: 2

Sol: 
\[ p = (2t + 1, -t - 1, t) \]
\[ Q = (s, 0, 3 - s) \]

\[ 2t + 1 - s = -t - 1 = t + s - 3 \]
\[ 3t - s = -2 \]
\[ 2t + s = 2 \]
\[ s = 2 \]
\[ Q = (2, 0, 1) \]

63. If \( \int x^5 e^{-x^2} \, dx = g(x)e^{-x^2} + c \), where \( c \) is a constant of integration then \( g(-1) \) is equal to:

1. \(-1\)
2. \(1\)
3. \(\frac{1}{2}\)
4. \(-\frac{5}{2}\)

Ans: 4

Sol: 
\[ x^2 = -t \]
\[ x \, dx = -\frac{1}{2} \, dt \]
\[ = \frac{1}{2} \int t^2 e^t \, dt \]
\[ = \frac{e^t}{2} (t^2 - 2t + 2) + c \]
\[ = -\frac{e^{-x^2}}{2} (x^4 + 2x^2 + 2) + c \]
\[ g(x) = -\frac{1}{2} (x^4 + 2x^2 + 2) \]
\[ g(-1) = -\frac{5}{2} \]

64. If both the mean and the standard deviation of 50 observations \( x_1, x_2, \ldots, x_{50} \) are equal to 16, then the mean of \( (x_1 - 4)^2, (x_2 - 4)^2, (x_{50} - 4)^2 \) is:

---

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65. The area (in sq.units) of the region bounded by the curves \( y = 2^x \) and \( y = |x+1| \), in the first quadrant is:

1. \( \frac{3}{2} - \frac{1}{\ln e} \)
2. \( \log_{e} + \frac{3}{2} \)
3. \( \frac{1}{2} \)
4. \( \frac{3}{2} \)

Ans: 1

Sol: 
\[
\frac{1}{50} \left( \sum X_i^2 - 8 \sum x_i + 16 \right) = 512 - 128 + 16 = 400
\]

66. Let \( a_1, a_2, a_3, \ldots \) be an A.P with \( a_6 = 2 \) then the common difference of this A.P, which maximizes the product \( a_1a_2a_3 \), is:

1. \( \frac{6}{5} \)
2. \( \frac{2}{3} \)
3. \( \frac{8}{5} \)
4. \( \frac{3}{2} \)

Ans: 3

Sol: 
\[
a_1 + 5d = 2
\]
\[
a_i(a_i + 3d)(a_i + 4d)
\]
\[
= (2 - 5d)(2 - 2d)(2 - d)
\]
\[
- f(d) = (d - 1)(d - 2)(5d - 2)
\]
\[
- f(d) = 5d^3 - 17d^2 + 16d - 4
\]
\[
15d^2 - 34d + 16 = 0
\]
\[
15d^2 - 10d - 24d + 16 = 0
\]
\[
(5d - 8)(3d - 2) = 0
\]
\[ d = \frac{8}{5} \]

\[-f''(d) = 30d - 34\]

67. Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium if the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number of beams is:

1. 1180
2. 170
3. 210
4. 190

Ans: 2

Sol: \( \text{beams} = \frac{20 \times 17}{2} = 170 \)

68. The locus of the centers of the circles, which touch the circle \( x^2 + y^2 = 1 \) externally, also touch the y-axis and lie in the first quadrant is:

1. \( y = \sqrt{1 + 2x}, x \geq 0 \)
2. \( x = \sqrt{1 + 2y}, y \geq 0 \)
3. \( y = \sqrt{1 + 4x}, x \geq 0 \)
4. \( x = \sqrt{1 + 4y}, y \geq 0 \)

Ans: 1

Sol: 
\[
x^2 + y^2 - 2x, x - 2y, y + y_i^2 = 0
\]
\[
x_i^2 + y_i^2 = (1 + x_i)^2
\]
\[
y_i^2 = 2x_i + 1
\]

69. If \( z \) and \( w \) are two complex numbers such that \( |zw| = 1 \) \( \arg(z) - \arg(w) = \frac{\pi}{2} \), then:

1. \( \bar{z}w = i \)
2. \( \bar{z}w = \frac{-1+i}{\sqrt{2}} \)
3. \( \bar{z}w = \frac{1-i}{\sqrt{2}} \)
4. \( \bar{z}w = -i \)

Ans: 1

Sol: \( Z = i, w = 1 \)

70. The negation of the Boolean expression \( \sim s \vee (\sim r \wedge s) \) is equivalent to:

1. \( s \vee r \)
2. \( r \)
3. \( \sim s \wedge \sim r \)
4. \( s \wedge r \)

Ans: 4

Sol: \( s \wedge (r \vee \sim s) \) \( (s \wedge r) \vee \phi = s \wedge r \)

71. If the line \( ax + y = c \), touches both the curves \( x^2 + y^2 = 1 \) and \( y^2 = 4\sqrt{2} \), then \( |c| \) is equal to
1. $\sqrt{2}$
2. 2.2
3. $\frac{1}{\sqrt{2}}$
4. $\frac{1}{2}$

Ans: 1

Sol: $y = -ax + c$

$c = \frac{-\sqrt{2}}{a}$

$c^2 = a^2 + 1$

$c^2 = \frac{2}{c^2} + 1$

$c^4 - c^2 - 2 = 0$

$c^2 = 2$

72. Let $y = y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} + y \tan x = 2x + x^2 \tan x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

such that if $y(0) = 1$, then

1. $y\left(\frac{\pi}{4}\right) - y\left(-\frac{\pi}{4}\right) = \pi - \sqrt{2}$
2. $y\left(\frac{\pi}{4}\right) - y\left(-\frac{\pi}{4}\right) = \sqrt{2}$
3. $y\left(\frac{\pi}{4}\right) + y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{2} + 2$
4. $y\left(\frac{\pi}{4}\right) + y\left(-\frac{\pi}{4}\right) = -\sqrt{2}$

Ans: 4

Sol: $\int \tan x \, dx = \sec x$

$y \sec x = \int \sec x \left(2x + x^2 \tan x\right)$

$= x^2 \sec x$

$y = x^2 + \cos x$

$y' = 2x - \sin x$

$y'\left(\frac{\pi}{4}\right) = \frac{\pi}{2} - \frac{1}{\sqrt{2}}$

$y'\left(-\frac{\pi}{4}\right) = -\frac{\pi}{2} + \frac{1}{\sqrt{2}}$

73. The tangent and normal to the ellipse $3x^2 + 5y^2 = 32$ at the point $P(2, 2)$ meet the x-axis at Q and R, respectively. Then the area (in sq.units) of the triangle PQR is:
1. \( \frac{68}{15} \)  
2. \( \frac{16}{3} \)  
3. \( \frac{14}{3} \)  
4. \( \frac{34}{15} \)  

Ans: 1  
Sol: \( 6x + 10yy' = 0 \)  
\[ y' = -\frac{3}{5} \]  
\[ \frac{4\left(1 + \frac{9}{25}\right)}{\frac{6}{5}} = \frac{4 \times 34}{25} \times \frac{5}{6} = \frac{68}{15} \]  

74. The angles A, B and C of a triangle ABC are in A.P and a:b=1: \( \sqrt{3} \). If c=4cm, then the area (in sq.cm) of this triangle is:  
1. \( 4\sqrt{3} \)  
2. \( 2\sqrt{3} \)  
3. \( \frac{4}{\sqrt{3}} \)  
4. \( \frac{2}{\sqrt{3}} \)  

Ans: 2  
Sol: \( A = 30^0, B = 60^0, C = 90^0 \)  
\( b = 2\sqrt{3} \)  
\[ \text{Area} = 2\sqrt{3} \]  

75. Lines are drawn parallel to the line \( 4x - 3y + 2 = 0 \), at a distance \( \frac{3}{5} \) from the origin. Then which one of the following points lies on any of these lines?  
1. \( \left(\frac{1}{4}, -\frac{1}{3}\right) \)  
2. \( \left(\frac{1}{4}, \frac{1}{3}\right) \)  
3. \( \left(-\frac{1}{4}, \frac{2}{3}\right) \)  
4. \( \left(-\frac{1}{4}, -\frac{2}{3}\right) \)  

Ans: 3  
Sol: \( 4x - 3y \pm 3 = 0 \)
76. Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is more than 99% is:

1. 8  
2. 2.5  
3. 3.7  
4. 4.6

Ans: 3

Sol: 
\[1 - p(x = 0) > \frac{99}{100}\]

\[\frac{1}{100} > \frac{1}{2^n}\]

\[n \geq 7\]

Minimum value n=7

77. If \(\cos^{-1} x - \cos^{-1} \frac{y}{2} = \alpha\), where \(-1 \leq x \leq 1, -2 \leq y \leq 2, x \leq \frac{y}{2}\) then for all \(x, y\)

\(4x^2 - 4xy\cos \alpha + y^2\) is equal to:

1. \(4\sin^2 \alpha\)  
2. \(2\sin^2 \alpha\)  
3. \(4\cos^2 \alpha + 2x^2y^2\)  
4. \(4\sin^2 \alpha - 2x^2y^2\)

Ans: 1

Sol: 
\(\cos^{-1} x = A, \cos^{-1} \frac{y}{2} = B\)

\(x = \cos A; \frac{y}{2} = \cos B\)

\(\sin A = \sqrt{1-x^2}, \sin B = \sqrt{1-\left(\frac{y}{2}\right)^2}\)

\(\frac{xy}{2} + \sqrt{(1+x^2)\left(1-\left(\frac{y^2}{4}\right)\right)} = \cos \alpha\)

\(1 - x^2 - \frac{y^2}{4} + \frac{x^2}{y^2} = \frac{x^2}{y^2} + \cos^2 \alpha - xy \cos \alpha\)

\(4 - 4x^2 - y^2 = 4\cos^2 \alpha - 4 + 4 \cos \alpha\)

78. If the tangent to the \(y = \frac{x}{\sqrt{x^2 + 3}}, x \in R, (x \neq \pm \sqrt{3})\), at a point \((\alpha, \beta) \neq (0,0)\) on it parallel to the line \(2x + 6y - 11 = 0\), then:

1. \(|2\alpha + 6\beta| = 11\)  
2. \(|6\alpha + 2\beta| = 19\)  
3. \(|6\alpha + 2\beta| = 9\)  
4. \(|2\alpha + 6\beta| = 19\)

Ans: 3
Sol: \( y(2x) + (x^2 - 3) \left( \frac{-1}{3} \right) = 1 \)
\[ 6\beta - \alpha = 0 \]
\[ (\alpha^2 - 3)\beta = 6\beta \]
\[ \alpha^2 = 9; \alpha = 3; \beta = \frac{1}{2} \]

79. Let \( \lambda \) be a real number of which the system of linear equations \( x + y + z = 6 \),
\( 4x + \lambda y - \lambda z = \lambda - 2 \)
\( 3x + 2y - 4z = -5 \) has infinitely many solutions. Then \( \lambda \) is a root of the
quadratic equation:
1. \( \lambda^2 + \lambda - 6 = 0 \)
2. \( \lambda^2 + 3\lambda - 4 = 0 \)
3. \( \lambda^2 - 3\lambda - 4 = 0 \)
4. \( \lambda^2 - \lambda - 6 = 0 \)
Ans: 4

Sol: \( x + y + z = 6 \).....(1)
\( 4x + \lambda y - \lambda z = \lambda - 2 \).....(2)
\( 3x + 2y - 4z = -5 \).....(3)

Eq(1)+(3)=(2)
\[ \lambda = 3 \]

80. If \( 5x + 9 = 0 \) is the direction of the hyperbola \( 16x^2 - 9y^2 = 144 \), then its corresponding
focus is:
1. \( \left( -\frac{5}{3}, 0 \right) \)
2. \( (-5, 0) \)
3. \( \left( \frac{5}{3}, 0 \right) \)
4. \((5,0)\)
Ans: 2

Sol: \( x = -\frac{9}{5} = -\frac{a}{e} = \frac{-3}{e} \)
\( e = \frac{5}{3} \) \( e = (-5, 0) \)

81. Let \( a, b \) and \( c \) be in G.P with common ratio \( r \), where \( a \neq 0 \) and \( 0 < r \leq \frac{1}{2} \). If \( 3a, 7b \) and
\( 15c \) are the first three terms of an A.P, then the 4th term of this A.P is
1. \( a \)
2. \( 5a \)
3. \( \frac{2}{3}a \)
4. \( \frac{7}{3}a \)
Ans: 1

Sol: \( 3a, 7ar, 15ar^2 \)
7r – 3 = 15r² – 7r
15r² – 14r + 3 = 0
15r² – 9r – 5r + 3 = 0
(3r – 1)(5r – 3) = 0

\[ x = \frac{1}{3} \]

3a, \( \frac{3a}{3}, \frac{5a}{3} \)

\[ 4 \text{ th term} = \frac{5a}{3} - \frac{2a}{3} = a \]

82. Let \( f(x) = \log_x (\sin x), (0 < x < \pi) \) \( g(x) = \sin^{-1}(e^{-x}), (x \geq 0) \). If \( \alpha \) is a positive real number such that \( a = (fog)'(\alpha) b = (fog)'(\alpha) \), then:

1. \( a\alpha^2 - b\alpha - a = 0 \)
2. \( a\alpha^2 + b\alpha - a = -2\alpha^2 \)
3. \( a\alpha^2 + b\alpha + a = 0 \)
4. \( a\alpha^2 - b\alpha - a = 1 \)

Ans: 4

Sol: \( b = -\alpha, a = -1 (fog)(x) = -x \)

83. If the plane \( 2x - y + 2z + 3 = 0 \) has the distances \( \frac{1}{3} \) and \( \frac{2}{3} \) units from the planes \( 4x - 2y + 4z + \lambda = 0 \) and \( 2x - y + 2z + \mu = 0 \), respectively then the maximum value of \( \lambda + \mu = \)

1. 1.5
2. 2.15
3. 3.9
4. 4.13

Ans: 4

Sol: \( \frac{\lambda - 6}{6} = \frac{1}{3}, \frac{\mu - 3}{3} = \frac{2}{3} \Rightarrow \lambda = 8; \mu = 5 \)

84. A spherical iron ball of radius 10cm is coated with a layer of ice of uniform thickness that melts a rate of 50cm³/min. When the thickness of the ice 5cm, then the rate at which the thickness (in cm/min) of the ice decreases, is:

1. \( \frac{5}{6\pi} \)
2. \( \frac{1}{18\pi} \)
3. \( \frac{1}{9\pi} \)
4. \( \frac{1}{36\pi} \)

Ans: 2

Sol: \( v = \frac{4\pi}{3}(10 + x)^3 \)
\[
\frac{dv}{dt} = 4\pi (10 + x)^2 \frac{dx}{dt}
\]
\[
\frac{du}{dt} = \frac{50}{4\pi \times 15 \times 15} = \frac{1}{18\pi}
\]

85. The integral \( \int_{\pi/6}^{\pi/3} \sec^{2/3} \cos ec^{4/3} \, dx = \)

1. \(3^{5/3} - 3^{1/3}\)
2. \(3^{7/6} - 3^{5/6}\)
3. \(3^{5/6} - 3^{2/3}\)
4. \(3^{4/3} - 3^{1/3}\)

Ans: 2

Sol:
\[
\int_{\pi/6}^{\pi/3} \frac{2}{3} \sec^2 u \frac{1}{(\tan u)^{4/3}} \, du
\]
\[
\int_{\pi/6}^{\pi/3} \frac{1}{4\sqrt{3}} \, dt
\]
\[
= \frac{1}{\sqrt{3}} [\tan^{-1}(\sqrt{3}) - \tan^{-1}(\sqrt{3})^{1/3}]
\]
\[
= \frac{1}{\sqrt{3}} [\pi/3 - (\pi/3)^{1/3}]
\]
\[
= 3^{7/6} - 3^{5/6}
\]

86. The distance of the point having position vector \(\mathbf{R} = 2\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}\) from the straight line passing through the point \((2,3,-4)\) and parallel to the vector \(6\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}\) is:

1. \(2\sqrt{13}\)
2. 2.7
3. 3.6
4. \(4\sqrt{3}\)

Ans: 2

Sol:
\[
\overrightarrow{AP} = (3,1,-10)
\]
\[
\sqrt{110}
\]
\[
\sqrt{61}
\]
\[
\sqrt{49}
\]
\[
\sqrt{61}
\]
\[
\sqrt{61}
\]
\[
\overrightarrow{AP}^2 = 110 - 61 = 49
\]
87. The number of real roots of the equation \(5 + |2^x - 1| = 2^x (2^x - 2)\) is:

1. 1  
2. 2.4  
3. 3.2  
4. 4.3

Ans: 1

Sol:
\[|2^n - 1| = t\]
\[5 + |t| = t^2 - 1\]
\[t^2 - |t| - 6 = 0\]
\[|t| = 3\]
\[2^n = 4 \text{ only}\]
\[2^n - 1 = \pm 3\]
\[2^n = 4\]

88. If \(\lim_{x \to 1} \frac{x^2 - ax + b}{x-1} = 5\), then \(a + b\) is equal to:

1. -4  
2. 2.1  
3. 3.5  
4. -7

Ans: 4

Sol:
\[1 - a + b = 0\]
\[2 - a = 5\]
\[a = -3\]
\[b = -4\]

89. The smallest natural number \(n\) such that the coefficient of \(x\) in the expansion of \(\left(x^2 + \frac{1}{x^3}\right)^n\) is \(n_{c3}\), IS:

1. 1.58  
2. 2.35  
3. 3.38  
4. 4.23

Ans: 3

Sol:
\[\frac{2n - 1}{5} = r\]
\[2n - 1 = 115\]
\[n = 58\]
\[n - 23 = \frac{2n + 1}{5}\]
\[
\begin{align*}
n - \frac{2n + 1}{5} &= 23 \\
3n + 1 &= 115; \quad n = \frac{114}{3} \\
n &= 38
\end{align*}
\]

90. The sum of \(1 + \frac{1^3 + 2^3}{1 + 2} + \frac{1^3 + 2^3 + 3^3}{1 + 2 + 3} + \ldots + \frac{1^3 + 2^3 + 3^3 + \ldots + 15^3}{1 + 2 + 3 + \ldots + 15} - \frac{1}{2}(1 + 2 + 3 + \ldots + 15) = \]

\[
\begin{align*}
&1.1240 & 2.1860 & 3.620 & 4.660 \\
\text{Ans:} & 3 \\
\text{Sol:} & \sum \frac{n(n + 1)}{2} = \sum \frac{n}{2} \\
&= \frac{1}{2} \left\{ \frac{15 \times 16 \times 31}{2} \right\} \\
&= 620
\end{align*}
\]