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IIT Academy., India

JEE - MAIN 2019

12th April 2019, Slot - 1

(9:30 am - 12:30 pm)

Question Paper

Solutions

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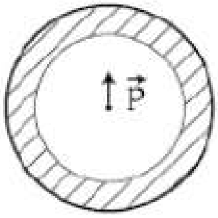
PHYSICS

1. Shown in the figure is a shell made of a conductor. It has inner radius a and outer radius b, and carries charge Q. At its centre is a dipole $\vec{p}$ as shown. In this case:



1. Surface charge density on the inner surface is uniform and equal to $\frac{(Q/2)}{4\pi a^2}$
2. Electric field outside the shell is the same as that of a point charge at the centre of the shell
3. Surface charge density on the inner surface of the shell is zero every where
4. Surface charge density on the outer surface depends on $|\vec{p}|$

Ans. 2

Sol. Electric field outside the shell is the same as that of a point charge at the centre of the shell ,From Gauss law as dipole does not create any charge inside

2. In a double slit experiment, when a thin film of thickness t having refractive index μ is introduced in front of one of the slits, the maximum at the centre of the fringe pattern shifts by one fringe width. The value of t is (λ is the wavelength of the light used).

1. $\frac{\lambda}{(2\mu-1)}$ 2. $\frac{2\lambda}{(\mu-1)}$ 3. $\frac{\lambda}{(\mu-1)}$ 4. $\frac{\lambda}{2(\mu-1)}$

Ans. 3

Sol: $(u-1)t = n\lambda$ Here $m=1$ $\therefore t = \frac{\lambda}{(\mu-1)}$

03. Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat of mixture at constant volume? (R = 8.3 J/mol K)

1.21.6 J/mol K 2.15.7 J/mol K 3.19.7 J/mol K 4.17.4 J/mol K

Ans. 4

Sol: $(C_V)_{mix} = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2} = \frac{2\times\frac{3R}{2} + 3\times\frac{5R}{2}}{2+3}$

$(C_V)_{mix} = \frac{R}{2}\left(\frac{6+15}{5}\right)$ $(C_V)_{mix} = \frac{21R}{10} = \frac{21\times 8.314}{10} = 17.45 J/molk$

04. An excited He^+ ion emits two photons in succession, with wavelengths 108.5 nm and 30.4 nm, in making a transition to ground state. The quantum number n, corresponding to its initial excited state is (for photon of wavelength λ, energy $E=\frac{1240\ eV}{\lambda(\text{in nm})}$.

1.n = 4 2. N = 5 3.N = 7 4.N = 6

Ans. 2

Sol: $E=\frac{1240}{108.5}=11.43 e.v$

$E_n=\frac{-4\times 13.612}{n^2} e.v$

If election Jumps from $n=5 \ to \ n=2$

$E=11.43 e.v$

$\therefore n=5$

05. A thin ring of 10 cm radius carries a uniformly distributed charge. The ring rotates at a constant angular speed of 40 π rad s^{-1} about its axis, perpendicular to its plane. If the magnetic field at its centre is 3.8×10^{-9} T, then the charge carried by the ring is close to $\left(\mu_0=4\pi\times 10^{-7}\ N/A^2\right)$

1. 3×10^{-5}C 2. 2×10^{-6}C 3. 4×10^{-5}C 4. 7×10^{-6}C

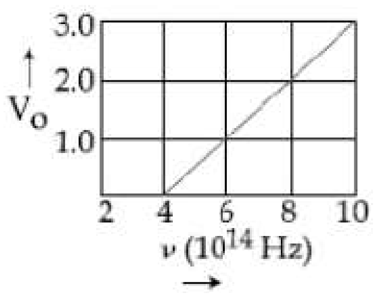
Ans: 1

Sol: $B=\frac{\mu_0 q\omega}{r 2\pi}$

$3.8\times 10^{-9}=\frac{4\pi\times 10^{-7}\times \rho\times 40\pi}{0.1\times 2\pi}$

$\rho=2\times 10^{-6} C$

06. The stopping potential V_0 (in volt) as a function of frequency (v) for a sodium emitter, is shown in the figure. The work function of sodium, from the data plotted in the figure, will be. (Given. Planck's constant (h) = 6.63×10^{-34} Js, electron charge e = 1.6×10^{-19}C)

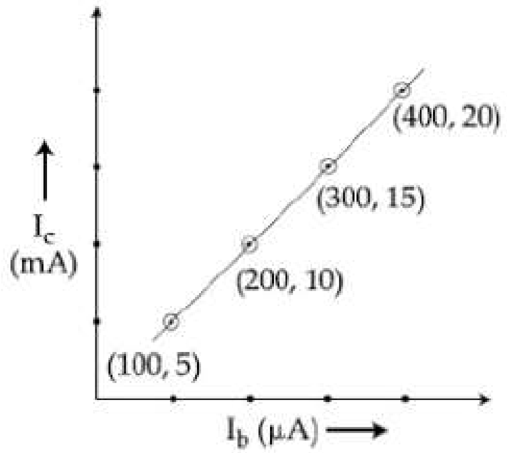


1.1.82 eV 2.2.12 eV 3.1.95 eV 4.1.66 eV

Ans: 4

Sol: $W = h\nu_0 \quad W = \frac{6.625\times10^{-34}\times4\times10^{14}}{1.6\times10^{-19}} \quad W = 1.66\,e.V$

07. **The transfer characteristic curve of a transistor, having input and output resistance $100\ \Omega$ and $100\ k\Omega$ respectively, is shown in the figure. The Voltage and Power gain, are respectively.**



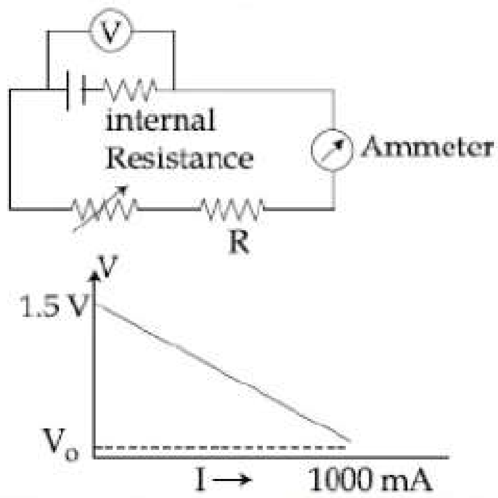
1. $5\times10^4, 2.5\times10^6$ **2.** $2.5\times10^4, 2.5\times10^6$ **3.** $5\times10^4, 5\times10^5$ **4.** $5\times10^4, 5\times10^6$

Ans: 1

Sol: $V_g = \frac{V_0}{V_I} = \frac{I_C R_0}{I_B R_I} \quad V_g = \frac{5\times10^{-3}}{100\times10^{-6}}\times\frac{100\times10^3}{100} \quad V_g = 5\times10^4 \quad P_g = \left(\frac{I_C}{I_b}\right)^2\times\frac{R_0}{R_I}$

$= \left(\frac{5\times10^{-3}}{100\times10^{-6}}\right)\times\frac{100\times10^3}{100} \quad p_g = \frac{10^6}{400}\times10^3 \quad p_g = 2.5\times10^6$

08. **To verify Ohm'slaw, a student connects the voltmeter across the battery as, shown in the figure. The measured voltage is plotted as a function of the current, and the following graph is obtained.**



If V_0 is almost zero, identify the correct statement.

1. The potential difference across the battery is 1.5 V when it sends a current of 1000 mA
2. The value of the resistance R is 1.5 Ω
3. The emf of the battery is 1.5 V and the value of R is 1.5 Ω
4. The emf of the battery is 1.5 V and its internal resistance is 1.5 Ω

Ans: 4

Sol: $V = E - Ir$

When $I = 0,\ V = E = 1.5V$

When $I = 1000\ mA,\ V = 0$

$\therefore\ \ 1.5 - 1.r = 0$

$r = 1.5\Omega$

09. A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of 0.70 ms^{-1} **with respect to the man. The speed of the man with respect to the surface is.**

1.0.28 ms^{-1} **2.0.47** ms^{-1} **3.0.14** ms^{-1} **4.0.20** ms^{-1}

Ans: 4

Sol: $V_1 + V_2 = 0.70$

$m_1V_1 = -m_2V_2$

$20(0.70 - V_2) = -50 \times V_2$

$14 - 20V_2 = 50V_2$

$-30V_2 = 14 \qquad V_2 = -0.47\,m/s$

10. A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 **are the values of the time taken by it to hit the target in two possible ways, the product** t_1t_2 **is**

1.R/2g **2.R/g** **3.R/4g** **4.2R/g**

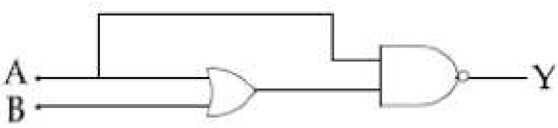
Ans: 4

Sol: $\frac{2u\sin\theta}{g} = t_1$

$\frac{2u\cos\theta}{g} = t_2$

$t_1t_2 = \frac{2R}{g}$

11. The truth table for the circuit given in the fig. is.



1. $\begin{vmatrix} A & B & Y \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}$ **2.** $\begin{vmatrix} A & B & Y \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix}$ **3.** $\begin{vmatrix} A & B & Y \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}$ **4.** $\begin{vmatrix} A & B & Y \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$

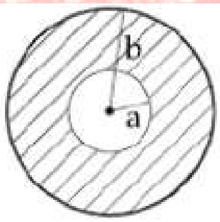
Ans: 2

Sol: $Y = \overline{A}.\overline{A+B}$

$Y = \overline{A}. + \overline{A+B}$

A	B	$Y = \overline{A}. + \overline{A+B}$
0	**0**	**1**
0	**1**	**1**
1	**0**	**1**
1	**1**	**0**

12. **A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$ then the radius of gyration of the disc about its axis passing through the centre is.**



1. $\frac{a+b}{3}$ **2.** $\sqrt{\frac{a^2+b^2+ab}{2}}$ **3.** $\frac{a+b}{2}$ **4.** $\sqrt{\frac{a^2+b^2+ab}{3}}$

Ans: 4

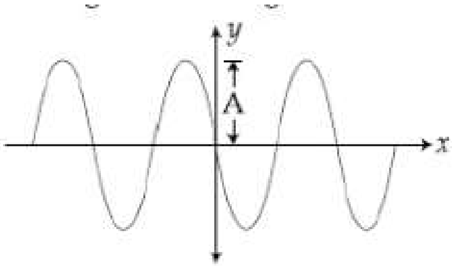
Sol: $mk^2 = \int_a^b (2\pi r dr)\frac{\sigma_0}{r} r^2$

$k^2 \int_a^b 2\pi r dr \frac{\sigma_0}{r} = \int (2\pi r dr)\frac{\sigma_0}{r} r^2$

$k^2 \; 2\pi\sigma.(b-a) = \frac{2\pi\sigma_0}{3}(b^3 - a^3)$

$k = \sqrt{\frac{a^2+b^2+ab}{3}}$

13. A progressive wave travelling along the positive x – direction is represented by $y(x,t) = A\sin(kx - \omega t + \phi)$. Its snapshot at t = 0 is given in the figure.



For this wave, the phase ϕ is

1. π 2. $\frac{\pi}{2}$ 3. $-\frac{\pi}{2}$ 4. 0

Ans: 1

Sol: $slope = \frac{dy}{dx} = AK\cos(kx - \omega t + \phi)$

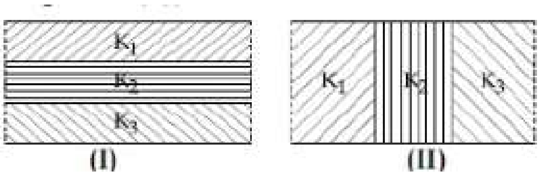
$at\ x = 0;\ t = 0,\ slope\ is$ ⊖ ve

$\therefore\ \ \phi = \pi$

Slope negative

14. Two identical parallel plate capacitors, of capacitance C each, have plates of area A, separated by a distance d. The space between the plates of the two capacitors, is filled with three dielectrics, of equal thickness and dielectric constants K_1, K_2 and K_3 .The first capacitor is filled as shown in fig. I, and the second one is filled as shown in fig II.

If these two modified capacitors are charged by the same potential V, the ratio of the energy stored in the two, would be (E_1 refers to capacitor (I) and E_2 to capacitor (II)).



1. $\frac{E_1}{E_2} = \frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{K_1K_2K_3}$

2. $\frac{E_1}{E_2} = \frac{K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

3. $\frac{E_1}{E_2} = \frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{9K_1K_2K_3}$

4. $\frac{E_1}{E_2} = \frac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

Ans: 4

Sol: $E_1 = \frac{1}{2}\frac{(K_1+K_2+K_3)}{3}C_0V^2$ $\qquad E_1 = \frac{1}{2}\frac{3K_1K_2K_3}{(K_1K_2+K_2K_3+K_3K_1)}C_0V^2$

$$\frac{E_1}{E_2} = \frac{(K_1+K_2+K_3)(K_1K_2+K_2K_3+K_3K_1)}{9(K_1K_2K_3)}$$

15. A person of mass M is, sitting on a swing of length L and swinging with an angular amplitude θ_0 .If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his centre of mass moves by a distance l(l<<L),is close to.

1. Mgl $\qquad$ **2.** $\mathrm{Mgl}\left(1-\frac{\theta_0^{\,2}}{2}\right)$ $\qquad$ **3.** $\mathrm{Mgl}\left(1+\theta_0^{\,2}\right)$ $\qquad$ **4.** $\mathrm{Mgl}\left(1-\theta_0^{\,2}\right)$

Ans: 3

Sol: Applying angular momentum conservation

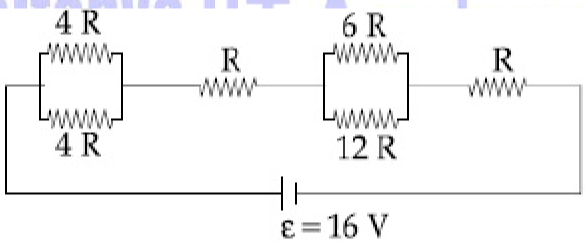
$$\mathrm{Mv}'(\mathrm{L}-\mathrm{l}) = \mathrm{mvL} \quad \Rightarrow \mathrm{v}' = \frac{\mathrm{vL}}{\mathrm{L}-\mathrm{l}}$$

Using Work-energy theorem

$$\mathrm{W_m} - \mathrm{Mgl} = \frac{1}{2}\mathrm{mv}'^2 - \frac{1}{2}\mathrm{m}$$

$$\Rightarrow \mathrm{W_m} = \mathrm{mgl}\left[1+\theta_0^2\right]$$

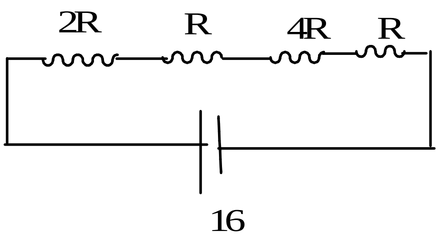
16. The resistive network shown below is connected to a D. C. source of 16 V. The power consumed by the network is 4 Watt. The value of R is.



1. 1Ω $\qquad$ **2.** 16Ω $\qquad$ **3.** 8Ω $\qquad$ **4.** 6Ω

Ans: 3

Sol:

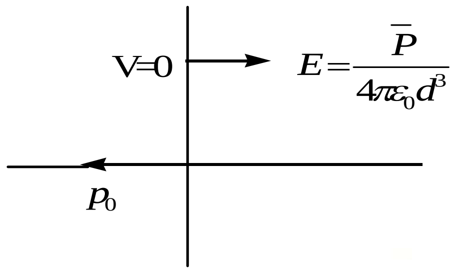


$$P = \frac{\varepsilon^2}{R} \Rightarrow 4 = \frac{256}{8R} \qquad R = 8\Omega$$

17. A point dipole $\vec{p} = -p_0 \hat{x}$ is kept at the origin. The potential and electric field due to this dipole on the y – axis at a distance d are, respectively. (Take V = 0 at indignity)

1. $\frac{|\vec{p}|}{4\pi\varepsilon_0 d^2}, \frac{-\vec{p}}{4\pi\varepsilon_0 d^3}$ 2. $\frac{|\vec{p}|}{4\pi\varepsilon_0 d^2}, \frac{\vec{p}}{4\pi\varepsilon_0 d^3}$ 3. $0, \frac{-\vec{p}}{4\pi\varepsilon_0 d^3}$ 4. $0, \frac{\vec{p}}{4\pi\varepsilon_0 d^3}$

Ans: 3



Sol:

18. An electromagnetic wave is represented by the electric field

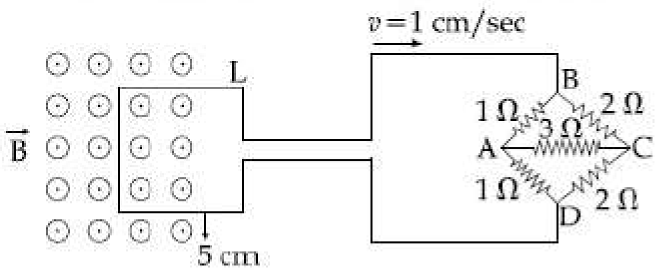
$\vec{E} = E_0 \hat{n} \sin\left[\omega t + (6y - 8z)\right]$. Taking unit vectors in x, y and z directions to be $\hat{i}, \hat{j}, \hat{k}$, the direction of propogation $\hat{s}$ is

1. $\hat{s} = \frac{-4\hat{k} + 3\hat{j}}{5}$ 2. $\hat{s} = \frac{4\hat{j} - 3\hat{j}}{5}$ 3. $\hat{s} = \frac{-3\hat{j} + 4\hat{k}}{5}$ 4. $\hat{s} = \frac{3\hat{j} - 4\hat{j}}{5}$

Ans: 3

Sol: $\overline{B} = E_0 \hat{A}\, 5n\left[\cos + (3i - 4j)\right]$ $\hat{S} = \frac{6\hat{i} - 8\hat{K}}{\sqrt{100}} = \frac{-3\hat{j} + 4\hat{k}}{5}$

19. The figure shows a square loop L of side 5 cm which is connected to a network of resistances. The whole setup is moving towards right with a constant speed of 1cm s^{-1}. At some instant, a part of L is in a uniform magnetic field of 1 T, perpendicular to the plane of the loop. If the resistance of L is 1.7 Ω ,the current in the loop at that instant will be close to.



1.170 μA 2.150 μA 3.115 μA 4. 60 μA

Ans: 1

Sol: $E = BlV = 1 \times 5 \times 10^{-2} \times 10^{-2} = 5 \times 10^{-4}\ volt$

$I = \frac{e}{R} = \frac{5 \times 10^{-4}}{(1.7 + 1.3)} = 1.66 \times 10^{-4}$ $I = 170\,\mu A$

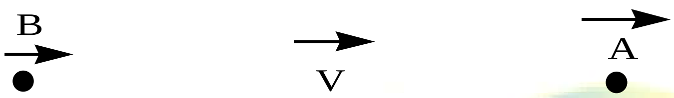
20. **A submarine (A) travelling at 18 km/hr is being chased along the line of its velocity by another submarine (B) travelling at 27 km/hr. B sends a sonar signal of 500 Hz to detect A and receives a reflected sound of frequency v. The value of v is close to. (Speed of sound in water = 1500 ms^{-1})**

1. 504 Hz **2.499 Hz** **3.507 Hz** **4.502 Hz**

Ans: 4

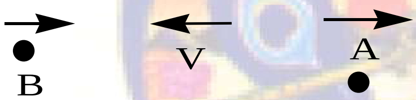
Sol: $V_A = 18 \times \frac{5}{18} = 5\,m/s \quad V = 1500\,m/s$

$V_B = 27 \times \frac{5}{18} = 7.5\,m/s \quad f = 500$



$v_A = \left(\frac{V - V_A}{V - V_B}\right) t \quad = \frac{1495}{1492.5} \times 500$

$v_A = 500.841 + 2$

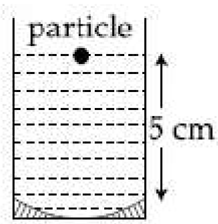


$v_B = v = \left(\frac{V + V_B}{V + V_A}\right) f$

$v = \frac{1507.5}{1505} \times 500.84$

$v = 502\,Hz$

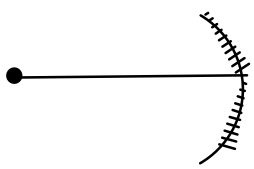
21. **A concave mirror has radius of curvature of 40 cm. It is at the bottom of a glass that has water filled up to 5 cm (see figure). It a small particle is floating on the surface of water, its image as seen, from directly above the glass, is at a distance d from the surface of water. The value of d is close to. (Refractive index of water = 1.33)**



1.11.7 cm **2.8.8 cm** **3.6.7 cm** **4.13.4 cm**

Ans: 2

Sol:



$\frac{1}{f} = \frac{1}{V} + \frac{1}{u}$

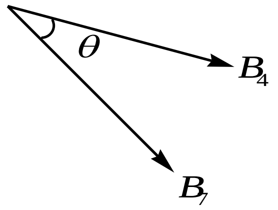
$$-\frac{1}{20}=\frac{1}{V}\quad \frac{1}{-\frac{4}{3}\times 5}\quad -\frac{1}{20}+\frac{3}{20}=\frac{1}{V}\quad \Rightarrow V=10\,cm\quad d=\frac{145}{\mu}=\frac{10+5}{1.33}=11.7\,cm$$

22. **A magnetic compass needle oscillates 30 times per minute at a place where the dip is** 45^0, and 40 **times per minute where the dip is** 30^0 **. If a**

1.3.6 **2.0.7** **3.2.2** **4.1.8**

Ans: 2

Sol:



$$n=\frac{1}{2\pi}\sqrt{\frac{B\cos\theta}{I}}\qquad \frac{n_1}{n_2}=\sqrt{\frac{B_1\cos\theta_1}{B_2\cos\theta_2}}=\frac{30}{40}\qquad \frac{B_1\times\frac{1}{\sqrt{2}}}{B_2\times\frac{\sqrt{3}}{2}}=\frac{9}{16}\qquad \frac{B_1}{B_2}=\frac{9}{16}\times\frac{\sqrt{3}}{2}\times\frac{\sqrt{2}}{1}=0.7$$

23. **The value of numerical aperture of the objective lens of a microscope is 1.25. If light of wavelength 5000** $\overset{0}{A}$ **is used, the minimum separation between two points, to be seen as distinct, will be.**

1.0.38 μm **2.0.48** μm **3.0.24** μm **4.0.12** μm

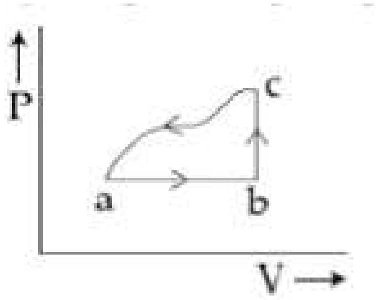
Ans: 3

Sol: $x_m=\frac{1.22x^2}{2.\sin x}$

$$=\frac{1.22\times 5000\times 10^{-10}}{2\times 1.25}$$

$$=0.24\,\mu m$$

24. **A sample of an ideal gas is taken through the cyclic process abca as shown in the figure. The change in the internal energy of the gas along the path ca is -180 J. The gas absorbs 250 J of heat along the path ab and 60 J along the path bc. The work done by the gas along the path abc is.**



1.120 J **2.140 J** **3.100 J** **4.130 J**

Ans: 4

Sol: $dQ = dU + dW$

$250 + 60 = 180 + dW$

$dW = 130\,J$

25. Which of the following combination has the dimension of electrical resistance (ε_0 is the permittivity of vacuum and μ_0 is the permeability of vacuum)?

1. $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ **2.** $\frac{\varepsilon_0}{\mu_0}$ **3.** $\frac{\mu_0}{\varepsilon_0}$ **4.** $\sqrt{\frac{\mu_0}{\varepsilon_0}}$

Ans: 4

Sol: $R = \left[ML^2T^{-3}A^{-2}\right]$

$\mu_0 = \left[MLT^{-2}A^{-2}\right]$

$E_0 = \left[M^{-1}L^{-3}T^4A^2\right]$

$R = \sqrt{\frac{\mu_0}{E_0}}$

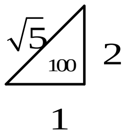
26. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed υ_0 then $\left(g = 10\text{ ms}^{-2}\right)$.

1. $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $\upsilon_0 = \frac{5}{3}\text{ms}^{-1}$ **2.** $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $\upsilon_0 = \frac{5}{3}\text{ms}^{-1}$

3. $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $\upsilon_0 = \frac{5}{3}\text{ms}^{-1}$ **4.** $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $\upsilon_0 = \frac{5}{3}\text{ms}^{-1}$

Ans: 1

Sol: $Tan\theta = 2$



$\cos\theta_0 = \frac{1}{\sqrt{5}}$

$\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$

$y = 2x - 9x^2$

$\frac{g}{2V_0^2\cos^2\theta} = 9$

$V_0^2\cos^2\theta = \frac{10}{18} \qquad = \frac{5}{9}$

$V_0\cos\theta = \frac{\sqrt{5}}{3} \qquad V_0 = \frac{5}{3}m/s$

27. **At** $40^{0}C$ **,a brass wire of 1 mm radius is hung from the ceiling. A small mass, M is hung from the free end of the wire. When the wire is cooled down from** $40^{0}C$ **to** $20^{0}C$ **it regains its original length of 0.2 m. The value of M is close to.**
(Coefficient of linear expansion and Young's modulus of brass are $10^{-5}/^{0}C$ and $10^{11}N/m^{2}$ **respectively ; g = 10 ms^{-2})**

1.9 kg **2.0.9 kg** **3.0.5 kg** **4.1.5 kg**

Ans: 1

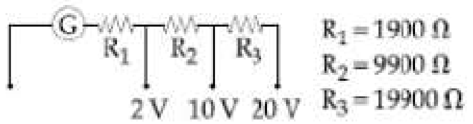
Sol: $Y A\alpha\Delta t = Mg$

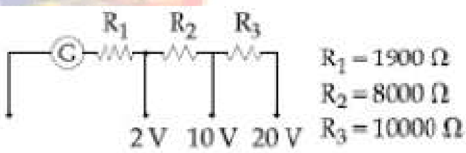
$$M = \frac{Y\pi r^{2}\times\Delta t}{g}$$

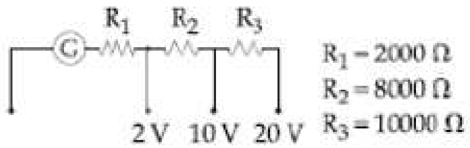
$$M = \frac{10^{11}\times\pi\times10^{-6}\times10^{-5}\times20}{10}$$

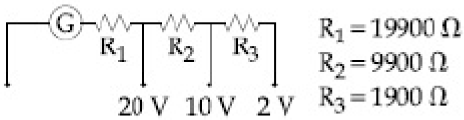
$M = 6.28\,ks$

28. **A galvanometer of resistance 100** Ω **has 50 divisions on its scale and has sensitivity of 20** $\mu A/$ **division. It is to be converted to a voltmeter with three ranges, of 0-2V, 0-10 V and 0-20V. The appropriate circuit to do so is:**

1. 

2. 

3. 

4. 

Ans: 2

Sol: $V = I(R+G)$

$2 = 10^{-3}(1900+100)$

$10 = 10^{-3}(9900+100)$

$20 = 10^{-3}(19900+100)$

29. **When** M_1 **gram of ice at** $-10^{0}C$ **(specific heat = 0.5 cal** $g^{-1^0}C^{-1}$**is added to** M_2 **gram of water at 50°C, finally no ice is left and the water is at 0°C. The value of latent heat of ice, in** $cal\,g^{-1}$**is:**

1. $\frac{5M_1}{M_1}-50$ **2.** $\frac{50M_2}{M_1}$ **3.** $\frac{5M_2}{M_1}-5$ **4.** $\frac{50M_2}{M_1}-5$

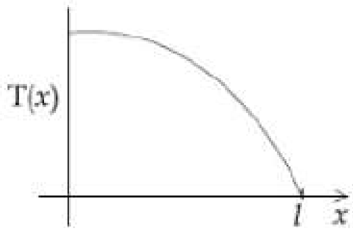
Ans: 4

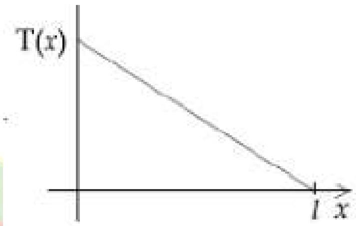
Sol: $M_1\times10\times0.5 + M_1L = M_2\times50$

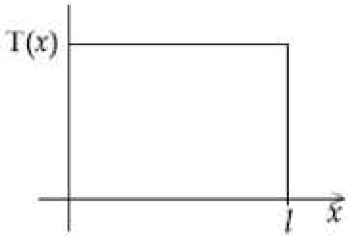
$$L = \frac{50M_2 - 5M_1}{M_1}$$

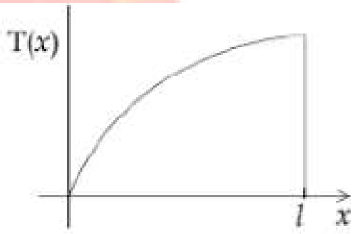
$$L = \frac{50M_2}{M_1} - 5$$

30. A uniform rod of length *l* is being rotated in a horizontal plane with a constant angular speed about an axis passing through one of its ends. If the tension generated in the rod due to rotation is T(x) at a distance x from the axis, then which of the following graphs depicts it most closely?

1. 

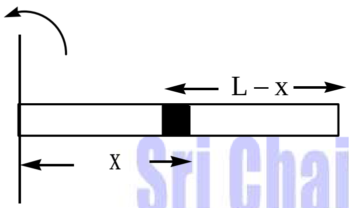
2. 

3. 

4. 

Ans: 1

Sol:



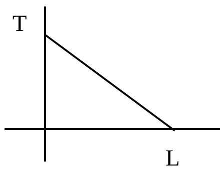
$$dT = dm x \omega^2$$

$$\partial T = \lambda dx\, x\, \omega^2$$

$$T = \lambda\omega^2 \int_x^{L-x} x\, dx$$

$$T = \frac{\lambda\omega^2}{2}\left[(L-x)^2 - x^2\right]$$

$$T = \frac{\lambda\omega^2}{2}\left[L^2 - 2Lx\right]$$



CHEMISTRY

31. In the following reaction: $xA \rightarrow yB$

$$\log_{10}\left[-\frac{d[A]}{dt}\right]=\log_{10}\left[\frac{d[B]}{dt}\right]+0.3010$$

'A' and 'B' respectively can be:

1. C_2H_4 and C_4H_8 **2. n – Butane and Iso – butane**

3. N_2O_4 and NO_2 **4.** C_2H_2 and C_6H_6

Ans: 1

Sol: $\frac{-d(x)}{dt}=+\frac{d(x)}{dt}\times\frac{x}{y}$

$$2C_2H_{4(x)}\rightarrow C_4H_{8(y)}$$

32. The basic structural unit of feldspar, zeolites, mica, and asbestos is:

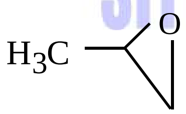
1. SiO_2 **2.** $(SiO_3)^{2-}$ **3.** $(SiO_4)^{4-}$ **4.** $\left(\begin{matrix}R\\|\\Si\\|\\R\end{matrix}-O\right)_n$ $(R=Me)$

Ans: 3

Sol: These are 3.0 silicates and basic unit is

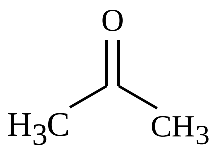
33. The major product of the following addition reaction is

$H_3C-CH=CH_2 \xrightarrow{Cl_2/H_2O}$

1. 

2. $H_3C-\underset{OH}{\underset{|}{CH}}-\underset{Cl}{\underset{|}{CH_2}}$

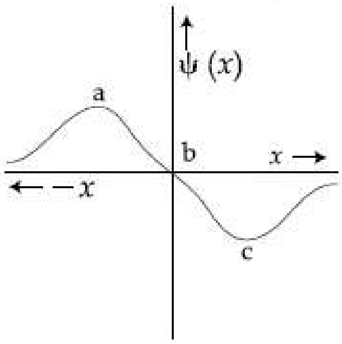
3. $CH_3-\underset{Cl}{\underset{|}{CH}}-\underset{OH}{\underset{|}{CH_2}}$

4. 

Ans: 2

Sol: $CH_3-CH=CH_2 \xrightarrow{cl_2/H_2o} CH_3-\underset{OH}{\underset{|}{CH}}-\underset{Cl}{\underset{|}{CH_2}}$

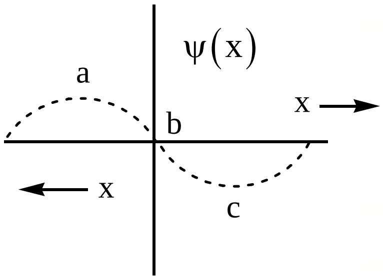
34. The electrons are more likely to be found:



1. in the region a and b **2. Only in the region a**
3. only in the region c **4. In the region a and c**

Ans: 4

Sol:



Probability chemistry $\psi^2 = a.c$

35. The correct sequence of thermal stability of the following carbonates is:

1. $BaCO_3 < SrCO_3 < CaCO_3 < MgCO_3$

2. $BaCO_3 < CaCO_3 < SrCO_3 < MgCO_3$

3. $MgCO_3 < CaCO_3 < SrCO_3 < BaCO_3$

4. $MgCO_3 < SrCO_3 < CaCO_3 < BaCO_3$

Ans: 3

Sol: $MgCO_3 < CaCO_3 < SrCO_3 < BaCO_3$

36. Enthalpy of sublimation of iodine is $24\,cal\,g^{-1}$ at $200°C$. **If specific heat of** $I_2(s)$ and $I_2(vap)$ **are 0.055 and** $0.031\,cal\,g^{-1}K^{-1}$ **respectively, then enthalpy of sublimation of iodine at 250°C in** $cal\,g^{-1}$ **is:**

1. 2.85 **2. 11.4** **3. 22.8** **4. 5.7**

Ans: 3

Sol:
$$\frac{\Delta H_2 - \Delta H_1}{T_2 - T_1} = \Delta C_P$$

$$\frac{\Delta H_2 - 24}{250 - 200} = (-0.024)$$

$$\Delta H_2 = 24 - 1.2 \quad = 22.8$$

37. **5 moles of** AB_2 **weigh** 125×10^{-3} kg **and 10 moles of** A_2B_2 **weigh** 300×10^{-3}kg**. The molar mass of** $A(M_A)$ **and molar mass of** $B(M_B)$ in $kg\,mol^{-1}$ **are:**

1. $M_A=10\times10^{-3}$ and $M_B=5\times10^{-3}$ **2.** $M_A=25\times10^{-3}$ and $M_B=50\times10^{-3}$

3. $M_A=50\times10^{-3}$ and $M_B=25\times10^{-3}$ **4.** $M_A=5\times10^{-3}$ and $M_B=10\times10^{-3}$

Ans: 4

Sol: Given the

(1) weight of 5 mol of AB $=125\times10^{-3}\ kg=125\ g$

or mass of 1 mol of $AB_2=25\,g$

(2) weight of 10 mol of $A_2B_2=300\times10^{-3}kg=300\,g$

mass of 1 mol of $A_2B_2=30\,g$

consider the molar mass as per molecular formula

$M_A+2M_B=25$ $2M_A+2M_B=30$

on solving; $M_A=5g$ and $M_B=10\,g$ or $M_A=5\times10^{-3}kg$ and $M_B=10\times10^{-3}kg$

38. **An element has a face – centred cubic (fcc) structure with a cell edge of a. The distance between the centres of two nearest tetrahedral voids in the lattice is:**

1. $\frac{3}{2}a$ **2.** $\sqrt{2}a$ **3. a** **4.** $\frac{a}{2}$

Ans: 4

Sol: In case of face-centred cubic structure, the distance between the centres of two nearest tetrahedral voids is $\frac{a}{2}$ which are parallel to edge of unit cell.

39. **Given:**

$Co^{3+}+e^{-1}\rightarrow Co^{2+}; E^\circ=+1.81V$

$Pb^{4+}+2e^{-}\rightarrow Pb^{2+}; E^\circ=+1.67V$

$Ce^{4+}+e^{-}\rightarrow Ce^{3+}; E=+1.61V$

$Bi^{3+}+3e^{-}\rightarrow Bi; E^\circ=+0.20V$

Oxidizing power of the species will increase in the order:

1. $Bi^{3+}<Ce^{4+}<Pb^{4+}<Co^{3+}$ **2.** $Co^{3+}<Ce^{4+}<Bi^{3+}<Pb^{4+}$

3. $Ce^{4+}<Pb^{4+}<Bi^{3+}<Co^{3+}$ **4.** $Co^{3+}<Pb^{4+}<Ce^{4+}<Bi^{3+}$

Ans: 1

Sol: Greater SRP, greater oxidising power

40. **The correct set of species responsible for the photochemical smog is:**

1. CO_2, NO_2, SO_2 and hydrocarbons **2.** N_2, NO_2 and hydrocarbons

3. NO, NO_2, O_3 and hydrocarbons **4.** N_2, O_2, O_3 and hydrocarbons

Ans: 3

Sol: Conceptual

41. The correct statement among the following is :

1. $(SiH_3)_3N$ is pyramidal and less basic than $(CH_3)_3N$

2. $(SiH_3)_3N$ is planar and less basic than $(CH_3)_3N$

3. $(SiH_3)_3N$ is planar and more basic than $(CH_3)_3N$

4. $(SiH_3)_3N$ is pyramidal and more basic than $(CH_3)_3N$

Ans: 2

Sol: The loss pair 'M' is donated to 3d orbital of Si^-

42. Which of the following is a thermosetting polymer?

1. PVC 2. Bakelite 3. Nylon 6 4. Buna – N

Ans: 2

Sol: Bakelite

43. An organic compound 'A' is oxidized with Na_2O_2 followed by boiling with HNO_3. The resultant solution is then treated with ammonium molybdate to yield a yellow precipitate. Based on above observation, the element present in the given compound is:

1. Fluorine 2. Sulphur 3. Nitrogen 4. Phosphorus

Ans: 4

Sol: $P \xrightarrow{Na_2O_2} Na_3PO_4 \xrightarrow{HNO_3} H_3PO_4 \xrightarrow{(NH_4)_2MoO_4} (NH_4)_3PO_4.12MoO_2 >$ yellow PPt

44. An example of a disproportionation reaction is:

1. $2KMnO_4 \rightarrow K_2MnO_4 + MnO_2 + O_2$

2. $2CuBr \rightarrow CuBr_2 + Cu$

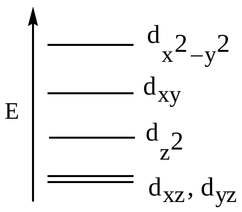
3. $2MnO_4^- + 10\,I^- + 16H^+ \rightarrow 2Mn^{2+} + 5I_2 + 8H_2O$

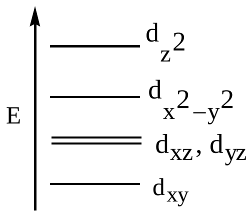
4. $2NaBr + Cl_2 \rightarrow 2NaCl + Br_2$

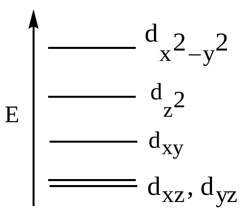
Ans: 2

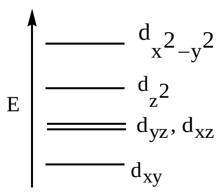
Sol: Cu^+ under go both oxidation reduction

45. Complete removal of both the axial ligands (along the z – axis) from an octahedral complex leads to which of the following splitting patterns? (relative orbital energies not on scale).

1. 

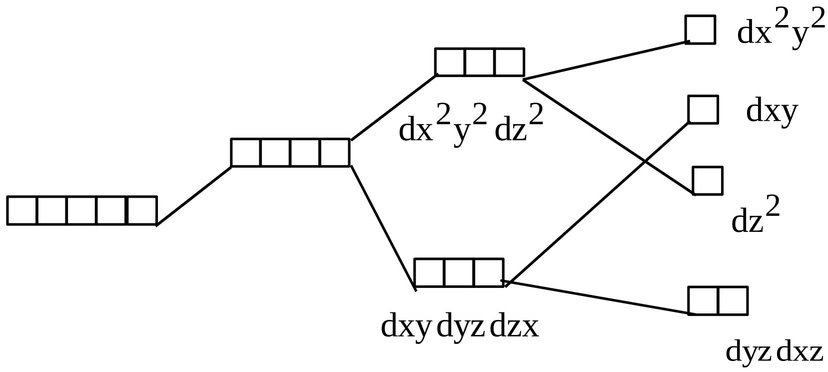
2. 

3. 

4. 

Ans: 1

Sol: **For square plane complex splitting**



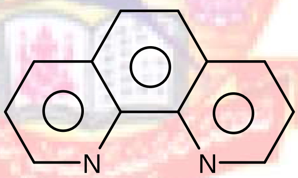
46. **The group number, number of valence electrons, and valencey of an element with atomic number 15, respectively, are:**

1. 16, 6 and 3 **2. 16, 5 and 2** **3. 15, 5 and 3** **4. 15, 6 and 2**

Ans: 3

Sol: ns^2np^3

47. **The complex ion that will lose its crystal field stabilization energy upon oxidation of its metal to +3 state is:**

(Phen = 

and

ignore pairing energy)

1. $\left[Co(phen)_3\right]^{2+}$ **2.** $\left[Zn(phen)_3\right]^{2+}$ **3.** $\left[Fe(phen)_3\right]^{2+}$ **4.** $\left[Ni(phen)_3\right]^{2+}$

Ans: 3

Sol: CFSE for $Fe^{+2} = -0.4(6) + 0.6(0)$

CFSE for $Fe^{+3} = -0.4(5) + 0.6(0)$

$= -2.0$ (decreases CFSE)

48. **But – 2 – ene on reaction with alkaline** $KMnO_4$ **at elevated temperature followed by acidification will give:**

1. $CH_3 - CH - CH - CH_3$ (with OH on each CH: $CH_3-CH(OH)-CH(OH)-CH_3$)

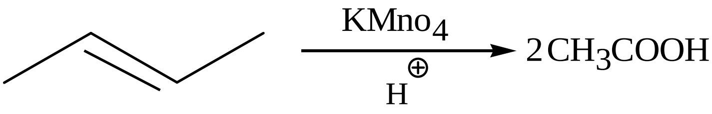
2. one molecule of CH_3CHO **and one molecule of** CH_3COOH

3. 2 molecules of CH_3CHO

4. 2 molecules of CH_3COOH

Ans: 4

Sol:



4.2 ,moles of CH_3COOH

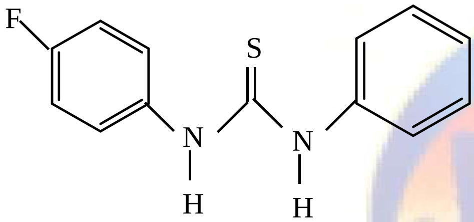
49. Glucose and Galactose are having identical configuration in all the positions except position.

1. C – 2 **2. C – 5** **3. C - 3** **4. C – 4**

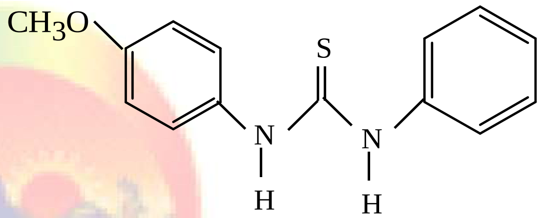
Ans: 4

Sol: C – 4 configuration is different

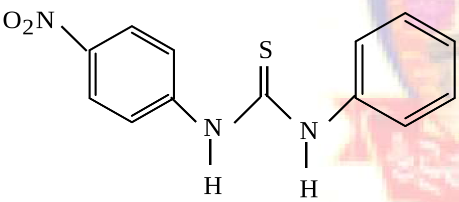
50. The increasing order of the pK_b **of the following compound is:**



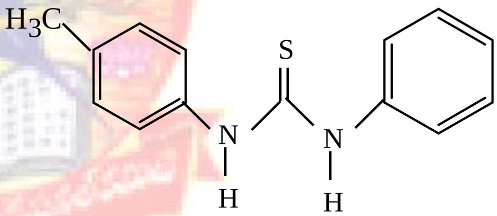
A.



B.



C.



D.

1. (C) < (A) < (D) < (B) **2. (B) < (D) < (A) < (C)**

3. (A) < (C) < (D) < (B) **4. (B) < (D) < (C) < (A)**

Ans: 1

Sol: B > D > A > C

51. What is the molar solubility of $Al(OH)_3$ **in 0.2 M NaOH solution? Given that, solubility product of** $Al(OH)_3 = 2.4\times10^{-24}$**:**

1. 3×10^{-19} **2.** 12×10^{-21} **3.** 3×10^{-22} **4.** 12×10^{-23}

Ans: 3

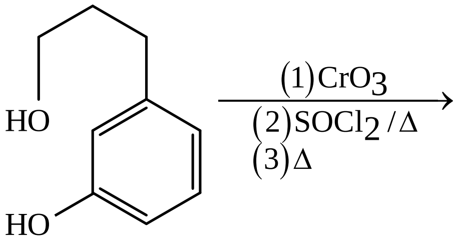
Sol: $Al(OH)_3(s) + aq \rightleftharpoons Al^{3+}_{(aq)} + 3OH^-_{(aq)}$

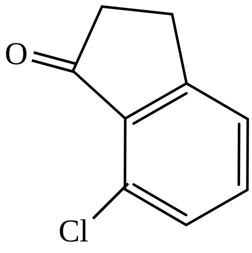
$$K_{sp} = 2.4\times10^{-24} = [Al^{3+}][OH^-]^3 = [Al^{3+}](0.2)^3$$

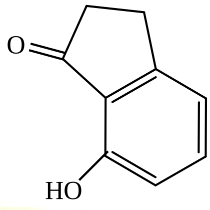
$$[Al^{3+}] = 3\times10^{-22}$$

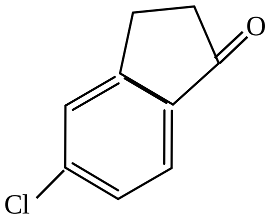
Thus solubility of $Al(OH)_3$ is 3×10^{-22}.

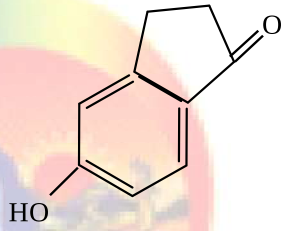
52. The major product of the following reaction is:



1. 

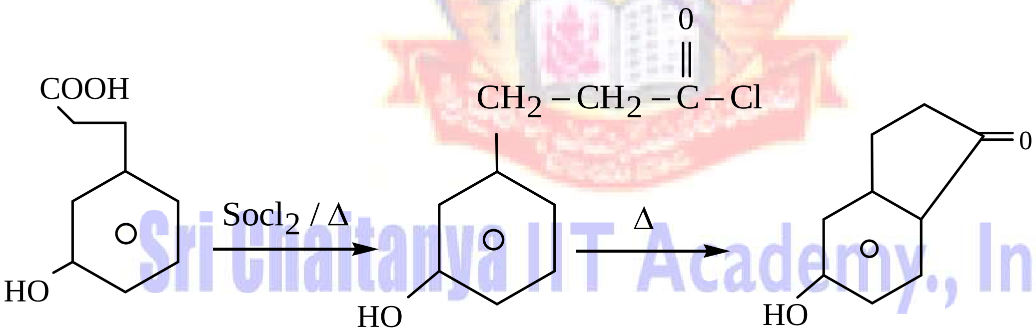
2. 

3. 

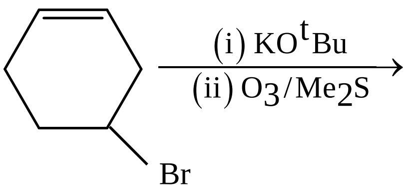
4. 

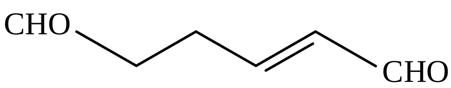
Ans: 4

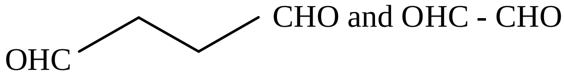
Sol:

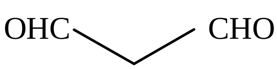


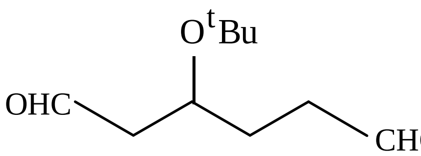
53. The major product(s) obtained in the following reaction is/are:



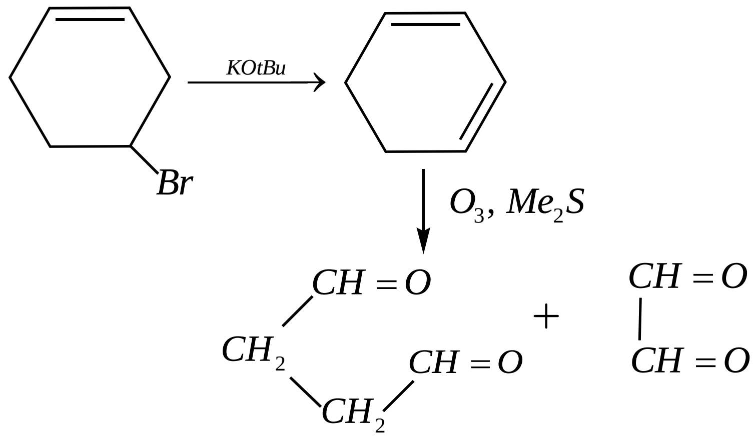
1. 

2. 

3. 

4. 

Ans: 2



Sol:

54. The metal that gives hydrogen gas upon treatment with both acid as well as base is:

1. magnesium 2. zinc 3. mercury 4. Iron

Ans: 2

Sol: $Zn + 2NaOH \rightarrow Na_2ZnO_2 + H_2$

$Zn + 2HCl \rightarrow ZnCl_2 + H_2$

55. An ideal gas is allowed to expand from 1L to 10 L against a constant external pressure of 1 bar. The work done is kJ is:

1. -0.9 2. -2.0 3. -9.0 4. +10.0

Ans: 1

SOL: $W = -Pdv \;\; = -1(9) = 9\,lt\,atm \;\; = \frac{9 \times 101.325}{1.01325 \times 1000} KJ = -0.9 \; KJ$

56. Which of the following statements is not true about RNA?

1. It usually does not replicate

2. It has always double stranded $\alpha - \text{helix}$ **structure**

3. It is present in the nucleus of the cell

4. It controls the synthesis of protein

Ans: 2

Sol: RNA is single Stranded

57. Peptization is a:

1. process of converting soluble particles to form colloidal solution

2. process of bringing colloidal molecule into solution

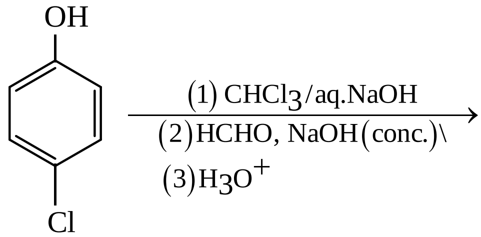
3. process of converting a colloidal solution into precipitate

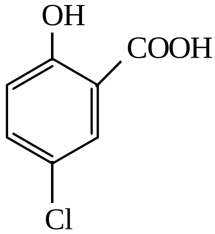
4. process of converting precipitate into colloidal solution

Ans: 4

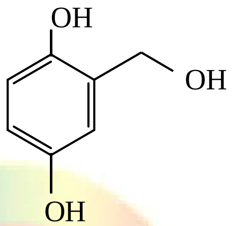
Sol: **Convergent of a freshly prepared precipitate into colloidal practical is known as petition**

58. **The major products of the following reaction are:**

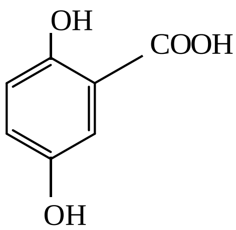


1. 

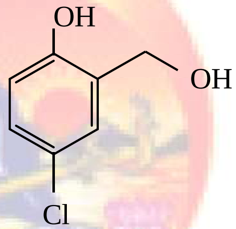
and Methanol

2. 

and Formic acid

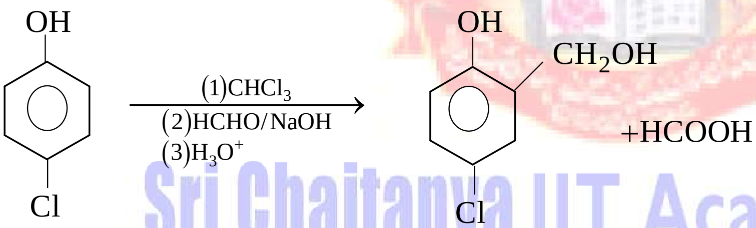
3. 

and Methanol

4. 

and Formic acid

Ans: **4**

Sol: 

59. **The mole fraction of a solvent in aqueous solution of a solute is 0.8. The molality** $\left(\text{in mol kg}^{-1}\right)$ **of the aqueous solution is:**

1. 13.88 **2.** 13.88×10^{-3} **3.** 13.88×10^{-2} **4.** 13.88×10^{-1}

Ans: **1**

Sol: $m=\frac{x}{1-x}\times\frac{1000}{\text{mwty solved}}$ **where** x_2 **mole fraction of solved** $=\frac{0.2}{0.8}\times\frac{1000}{18}=13.88$

60. **The ideal of froth floatation method came from a person X and this method is related to the process Y of ores. X and Y, respectively, are:**

1. washer man and reduction
2. Fisher women and concentration
3. washer women and concentration
4. Fisher man and reduction

Ans: **3**

Sol: **Washer woman and concentration**

MATHEMATICS

61. **The value of** $\sin^{-1}\left(\frac{12}{13}\right)-\sin^{-1}\left(\frac{3}{5}\right)$ **is equal to:**

1. $\pi-\cos^{-1}\left(\frac{33}{65}\right)$ **2.** $\frac{\pi}{2}-\cos^{-1}\left(\frac{9}{65}\right)$ **3.** $\frac{\pi}{2}-\sin^{-1}\left(\frac{56}{65}\right)$ **4.** $\pi-\sin^{-1}\left(\frac{63}{65}\right)$

Ans: 3

Sol: $\sin^{-1}\left(\frac{12}{13}.\frac{4}{5}-\frac{3}{5}.\frac{5}{13}\right)$

$=\frac{\pi}{2}-\sin^{-1}\left(\frac{56}{65}\right)$

62. **If the normal to the ellipse** $3x^2+4y^2=12$ **at a point P on it is parallel to the line, 2x + y = 4 and the tangent to the ellipse at P passes through Q (4, 4) then PQ is equal to:**

1. $\frac{\sqrt{61}}{2}$ **2.** $\frac{\sqrt{221}}{2}$ **3.** $\frac{\sqrt{157}}{2}$ **4.** $\frac{5\sqrt{5}}{2}$

Ans: 4

Sol: $\frac{a^2y}{b^2x}=\frac{a}{b}\tan\theta=-2$

$a=2, b=\sqrt{2}$, $\tan\theta=-\sqrt{3}$, $\sin\theta=\frac{\sqrt{3}}{2}, \cos\theta=-\frac{1}{2}$

$\therefore P\left(2\cos\theta,\sqrt{3}\sin\theta\right)=\left(-1,3/2\right)$

$Q(4,4)\Rightarrow PQ=\frac{5\sqrt{5}}{2}$

63. **If the data** $x_1, x_2,.....x_{10}$ **is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all of these is 2,000; then the standard deviation of this data is:**

1. 2 **2.** $2\sqrt{2}$ **3.** 4 **4.** $\sqrt{2}$

Ans: 1

Sol: $\sum x_i=11\times4+16\times6$

= 44+96 = 140

$\sum x_i^2=2,000$

S.D $=\sqrt{\frac{\sum x_i^2}{10}-\left(\frac{\sum x_i}{10}\right)^2}$

= 2

64. The equation $y=\sin x\sin(x+2)-\sin^2(x+1)$ represents a straight line lying in:

1. third and fourth quadrants only
2. Second and third quadrants only
3. first, second and fourth quadrants
4. First, third and fourth quadrants

Ans: 1

Sol: $y=\sin x\sin(x+2)-\sin^2(x+1)$

$=\frac{1}{2}[\cos 2-\cos(2x+2)]-\frac{1-\cos(2x+2)}{2}$

$=\frac{\cos 2-1}{2}=-\sin^2 1$

65. The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is:

1. 2^{20} 2. $2^{20}+1$ 3. 2^{21} 4. $2^{20}-1$

Ans: 1

Sol: No. of ways

$=21_{C_{10}}+21_{C_9}+........+21_{C_0}=S$

$S+S=2^{21}$

$S=2^{20}$

66. For $x\in R$, let [x] denote the greatest integer $\le x$, then the sum of the series

$\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+....+\left[-\frac{1}{3}-\frac{99}{100}\right]$ is:

1. -133 2. -153 3. -135 4. -131

Ans: 1

Sol: -1-1-.....-1-2-2....-2

67 times 33 times

=-67-66=-133

67. Let $f:R\to R$ be a continuously differentiable function such that f(2) = 6 and

$f'(2)=\frac{1}{48}$. If $\int_6^{f(x)}4t^3dt=(x-2)g(x)$, then $\lim\limits_{x\to 2}g(x)$ is equal to:

1. 24 2. 18 3. 36 4. 12

Ans: 2

Sol: $\int\limits_6^{f(x)}4t^2d(x-2)g(x)$

$$\Rightarrow g(x)=\frac{\int_{6}^{f(x)} 4t^2 dt}{x-2}$$

$$\underset{x\to 2}{\text{lt}}\ g(x)=\underset{x\to 2}{\text{lt}}\ \frac{4(f(x))^3.f'(x)}{1}$$

$$=4.6^3.\frac{1}{48}$$

= 18

68. Consider the differential equation, $y^2dx+\left(x-\frac{1}{y}\right)dy=0$. If value of y is 1 when x = 1, then the value of x for which y = 2, is:

1. $\frac{3}{2}-\frac{1}{\sqrt{e}}$ **2.** $\frac{5}{2}+\frac{1}{\sqrt{e}}$ **3.** $\frac{3}{2}-\sqrt{e}$ **4.** $\frac{1}{2}+\frac{1}{\sqrt{e}}$

Ans: 1

Sol: $y^2dx+\left(x-\frac{1}{y}\right)dy=0$

$$\Rightarrow \frac{dx}{dy}+\frac{x}{y^2}=\frac{1}{y^3}$$

$$IF=e^{\int \frac{1}{y^2}dy}=e^{\frac{-1}{y}}$$

Sol is $xe^{\frac{-1}{y}}=\int e^{\frac{-1}{y}}.\frac{1}{y^3}dy+c$

$$\Rightarrow xe^{\frac{-1}{y}}=-\left[e^{\frac{-1}{y}}\left(\frac{-1}{y}\right)-e^{\frac{-1}{y}}\right]+c$$

$$\Rightarrow x=\frac{1}{y}+1+c\,e^{\frac{1}{y}}$$

$$x=1,\ y=1\Rightarrow 1=1+1+ce$$

$$\Rightarrow c=\frac{-1}{e}$$

$$\therefore\ x=\frac{1}{y}+1-\frac{1}{e}e^{\frac{1}{y}}$$

$$y = 2 \Rightarrow x = \frac{1}{2} + 1 - \frac{1}{e}e^{\frac{1}{2}}$$
$$= \frac{3}{2} - \frac{1}{\sqrt{e}}$$

69. For $x \in \left(0, \frac{3}{2}\right)$, let $f(x) = \sqrt{x}$, $g(x) = \tan x$ **and** $h(x) = \frac{1-x^2}{1+x^2}$. **If** $\phi(x)((hof)og)(x)$ **then** $\phi\left(\frac{\pi}{3}\right)$ **is equal to:**

1. $\tan\frac{5\pi}{12}$ **2.** $\tan\frac{11\pi}{12}$ **3.** $\tan\frac{\pi}{12}$ **4.** $\tan\frac{7\pi}{12}$

Ans: 2

Sol: $\phi(x) = \frac{1-\tan x}{1+\tan x} = \tan\left(\frac{\pi}{4} - x\right)$

$\phi\left(\frac{\pi}{3}\right) = \tan\frac{\pi}{12}$

$= \tan\frac{11\pi}{12}$

70. If the line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ **intersects the plane 2x + 3y – z + 13 = 0 at a point P and the plane 3x + y + 4z = 16 at a point Q, then PQ is equal to:**

1. $\sqrt{14}$ **2.** $2\sqrt{7}$ **3. 14** **4.** $2\sqrt{14}$

Ans: 4

Sol: $P = (3t+2, 2t-1, -t+1)$ lies on $2x + 3y - z + 13 = 0$

$\Rightarrow 6t + 4 + 6t - 3 + t - 1 + 13 = 0$

$\Rightarrow 13t = -13$

$\Rightarrow t = -1$

$P = (-1, -3, 2)$

$Q = (3t+2, 2t-1, -t+1)$ lies on $3x + y + 4z = 16$

$\Rightarrow 9t + 6 + 2t - 1 - 4t + 4 = 16$

$\Rightarrow 7t = 7 \quad \Rightarrow t = 1$

$Q = (5, \}0)$

$\therefore PQ = \sqrt{36+16+4} = \sqrt{56} = 2\sqrt{14}$

71. The coefficient of x^{18} **in the product** $(1+x)(1-x)^{10}(1+x+x^2)^9$ **is:**

1. 126 **2. -126** **3. -84** **4. 84**

Ans: 4

Sol: $(1+x)(1-x)^{10}\left(1+x+x^2\right)^9$

$=(1+x)(1-x)\left(1-x^3\right)^9$

$=\left(1-x^2\right)\left(1-c_1x^3+c_2x^6-c_3x^9+......\right)$

coef $x^{18}=9_{c_6}=84$

72. The integral $\int\frac{2x^3-1}{x^4+x}dx$ is equal to: (Here C is a constant of integration)

1. $\frac{1}{2}\log_e\frac{\left(x^3+1\right)^2}{\left|x^3\right|}+C$
2. $\log_e\frac{\left|x^3+1\right|}{x^2}+C$
3. $\frac{1}{2}\log_e\frac{\left|x^3+1\right|}{x^2}+C$
4. $\log_e\left|\frac{x^3+1}{x}\right|+C$

Ans: 4

Sol: $\int\frac{2x^3-1}{x^4+x}dx$

$=\int\frac{2\left(x^3+1\right)-3}{x\left(x^3+1\right)}dx$

$=\int\frac{2}{x}-\frac{3x^2}{x^3\left(x^3+1\right)}dx$

$=\int\frac{2}{x}dx-\int\left(\frac{1}{x^3}-\frac{1}{x^3+1}\right)dx^3$

$=2\log|x|-\log\left|x^3\right|+\log\left|x^3+1\right|+c$

$=\log\frac{\left|x^3+1\right|}{|x|}+c$

73. If the volume of parallelopiped formed by the vectors $\hat{i}+\lambda\hat{j}+\hat{k},\ \hat{j}+\lambda\hat{k}$ and $\lambda\hat{i}+\hat{k}$ is minimum, then λ is equal to:

1. $-\sqrt{3}$
2. $\sqrt{3}$
3. $\frac{1}{\sqrt{3}}$
4. $-\frac{1}{\sqrt{3}}$

Ans: 3

Sol: $V=\begin{vmatrix}1 & \lambda & 1\\ 0 & 1 & \lambda\\ \lambda & 0 & 1\end{vmatrix}$

$V(\lambda)=\lambda^3-\lambda+1$

$V'(\lambda)=3\lambda^2-1$

let $V'(\lambda)=0$

$\lambda=\pm\frac{1}{\sqrt{3}}$

$V''(\lambda)=6\lambda$

$V''\left(\frac{1}{\sqrt{3}}\right)=\frac{6}{\sqrt{3}}>0$

$\lambda=\frac{1}{\sqrt{3}}$

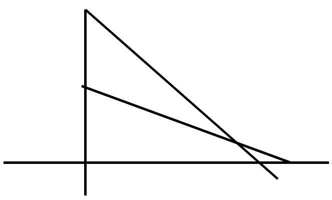
74. A 2m ladder leans against a vertical wall. If the top of the ladder begins to slide down the wall at the rate 25cm/sec., then the rate (in cm/sec.) at which the bottom of the ladder slides away from the wall on the horizontal ground when the top of the ladder is 1m above the ground is:

1. $\frac{25}{3}$ **2. 25** **3.** $25\sqrt{3}$ **4.** $\frac{25}{\sqrt{3}}$

Ans: 4

Sol: $x^2+y^2=4$

$zx\frac{dx}{dt}+2y\frac{dy}{dt}=0$



$x\frac{dx}{dt}=-y\frac{dy}{dt}$

$\sqrt{3}\frac{dx}{dt}=(-1)25$

$\frac{dx}{dt}=\frac{25}{\sqrt{3}}m/sec$

$y=1\Rightarrow x^2=3$

$\Rightarrow x=\sqrt{3}$

75. The equation $|z-i|=|z-1|, i=\sqrt{-1}$, represents:

1. a circle of radius 1
2. a circle of radius $\frac{1}{2}$
3. the line through the origin with slope – 1
4. the line through the origin with slope 1.

Ans: 4

Sol: $|z-i|=|z-1|$

$x^2+(y-1)^2=(x-1)^2+y^2$

$x^2+y^2-2y+1=x^2-2x+1+y^2$

$-2y=-2x$

$x-y=0$

$y=x$

76. If $\int_0^{\frac{\pi}{2}} \frac{\cot x}{\cot x+\operatorname{cosec} x} dx=m(\pi+n)$, then m.n is equal to:

1. -1
2. $-\frac{1}{2}$
3. 1
4. $\frac{1}{2}$

Ans: 1

Sol: $I=\int_0^{\frac{\pi}{2}} \operatorname{cosec} x \cot x-\cot^2 x$

$I=\int_0^{\frac{\pi}{2}}\left(\operatorname{cosec} x \cot x-\operatorname{cosec}^2 x+1\right) dx$

$=(-\operatorname{cosec} x+\cot x+x)_0^{\frac{\pi}{2}}$

$=\frac{\pi}{2}-1$

$=\frac{1}{2}(\pi-2)$

$m \times n=+\frac{1}{2} \times(-2)$

$=-1$

$m=\frac{1}{2}, \quad n=-2$

77. If $e^y + xy = e$, the ordered pair $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right)$ at x = 0 is equal to:

1. $\left(-\frac{1}{e}, -\frac{1}{e^2}\right)$ 2. $\left(\frac{1}{e}, -\frac{1}{e^2}\right)$ 3. $\left(-\frac{1}{e}, \frac{1}{e^2}\right)$ 4. $\left(\frac{1}{e}, \frac{1}{e^2}\right)$

Ans: 3

Sol: $y' = \frac{-y}{e^y + x}$

$y'(0) = -\frac{1}{e}$

$y'' = \frac{-\left(e^4 + x\right)y' + y\left(e^3y' + 1\right)}{\left(e^4 + x\right)^2}$

$y'' = \frac{1}{e^2}$

$(y', y'') = \left(\frac{-1}{e}, \frac{1}{e^2}\right)$

78. If $B = \begin{bmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{bmatrix}$ is the inverse of a 3×3 matrix A, then the sum of all values of α for which det (A) + 1 = 0, is:

1. -1 2. 2 3. 1 4. 0

Ans: 3

Sol: $\det A = \frac{1}{|B|}$

$\frac{1}{|B|} + 1 = 0 \Rightarrow |B| = -1$

$\Rightarrow 2\alpha^2 - 2\alpha - 25 + 1 = 0$

$\Rightarrow \alpha = 4, -3$

79. Let a random variable X have a binomial distribution with mean 8 and variance 4. If $P(X \le 2) = \frac{k}{2^{16}}$, then k is equal to:

1. 1 2. 121 3. 17 4. 137

Ans: 4

Sol: np = 8, npq = 4 $\Rightarrow q = \frac{1}{2}, p = \frac{3}{2} n = 16$

$P(x \le 2) = P(x=0) + P(x=1)P(x=2)$

$= \left(16_{C_0} + 16_{C_1} + 16_{C_2}\right)\dfrac{1}{2^{16}}$

$= \dfrac{1+16+120}{2^{16}} = \dfrac{137}{2^{16}}$

80. **Let** $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ **and** $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ **be two vectors. If a vector perpendicular to both vectors** $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ **has the magnitude 12 then one such vector is:**

1. $4\left(2\hat{i} + 2\hat{j} + \hat{k}\right)$ **2.** $4\left(-2\hat{i} - 2\hat{j} + \hat{k}\right)$ **3.** $4\left(2\hat{i} + 2\hat{j} - \hat{k}\right)$ **4.** $4\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$

Ans: 4

Sol: $\bar{a} + \bar{b} = (4,4,0), \bar{a} - \bar{b} = (2,0,4)$

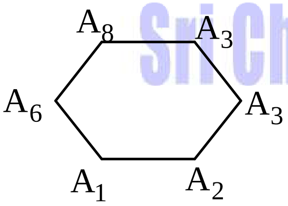
$\dfrac{\bar{a} + \bar{b} = \left(\bar{a} - \bar{b}\right)}{\left|\bar{a} + \bar{b} \times \bar{a} - \bar{b}\right|} = \bar{u} = \dfrac{2\hat{i} - 2\hat{j} - 8}{3}$

$\therefore 12\bar{u} = 4\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$

81. **If three of the six vertices of a regular hexagon are chosen at random, then the probability that the triangle formed with these chosen vertices is equailateral is:**

1. $\dfrac{3}{20}$ **2.** $\dfrac{1}{5}$ **3.** $\dfrac{1}{10}$ **4.** $\dfrac{3}{10}$

Ans: 3



Sol:

(A_1, A_3, A_5)

(A_2, A_4, A_6)

Form Equilateral triangle

$\dfrac{2}{6_{C_3}} = \dfrac{2}{20} = \dfrac{1}{10}$

82. **If m is the minimum value of k for which the function** $f(x) = x\sqrt{kx - x^2}$ **is increasing in the interval [0, 3] and M is the maximum value of f in [0, 3] when k = m, then the ordered pair (m, M) is equal to:**

1. $\left(4, 3\sqrt{3}\right)$ **2.** $\left(3, 3\sqrt{3}\right)$ **3.** $\left(4, 3\sqrt{2}\right)$ **4.** $\left(5, 3\sqrt{6}\right)$

Ans: 1

Sol: $f_k(x) = x\sqrt{kx - x^2}$

$f_k(3) = 3\sqrt{3k-9}$

$f_k^1(x) = 0$ **and for function to be ↑, the minimum value of k = 4 where x = 3**

$\therefore m = 4,\ m = 3\sqrt{12-9} = 3\sqrt{3}$

$(m, m) = (4, 3\sqrt{3})$

83. **If the area (in sq.units) of the region** $\{(x,y): y^2 \le 4x, x+y \le 1,\ x \ge 0,\ y \ge 0\}$ **is** $a\sqrt{2}+b$**, then a – b is equal to**

1. $\frac{10}{3}$ **2.** $-\frac{2}{3}$ **3. 6** **4.** $\frac{8}{3}$

Ans: 3

Sol: $y^2 = 4x$

$x + y = 4$

$(1-x)^2 = 4x$

$x^2 - 2x + 1 - 4x = 0$

$x^2 - 6x + 1 = 0$

$x = \frac{6 \pm \sqrt{36-4}}{2}$

$3 = 2\sqrt{2}$

$\int_0^{3-2\sqrt{2}} 2\sqrt{x} + \frac{1}{2}(1-3+2\sqrt{2})^2$

$2\left[x\sqrt{x}.\frac{2}{3}\right]_0^{3-2\sqrt{2}} + \frac{1}{2}(2\sqrt{2}-2)^2$

$\frac{4}{3}\left[(\sqrt{2}-1)^3\right] + 2(\sqrt{2}-1)^2$

$(\sqrt{2}-1)^2\left[\frac{4}{3}(\sqrt{2}-1)+2\right]$

$(3-2\sqrt{2})\left[\frac{4\sqrt{2}+2}{3}\right]$

$\frac{12\sqrt{2}\left[4\sqrt{2}+2\right]}{3}$

$2\frac{8\sqrt{2}-10}{3}$ $\quad a=\frac{8}{3},\ b=\frac{-10}{3}$

$a-b=6$

84. Let S_n denote the sum of the first n terms of an A.P.. If $S_4=16$ and $S_6=-48$, then S_{10} is equal to:

1. -260 **2. -320** **3. -380** **4. -410**

Ans: 2

Sol: $\frac{4}{2}[2a+3d]=16,$

$\frac{6}{2}[2a+5d]=-48,$

d = -12 , a = 44

$S_{10}=\frac{10}{2}(44-108)=-320$

85. The number of solutions of the equation

$1+\sin^4 x=\cos^2 3x,\ x\in\left[-\frac{5\pi}{2},\frac{5\pi}{2}\right]$ **is:**

1. 7 **2. 5** **3. 3** **4. 4**

Ans: 2

Sol: $1+\sin^4 x=\cos^2 3x$

$\sin^2 3x+\sin^4 x=0$

$\sin 3x=0$ and $\sin x=0$

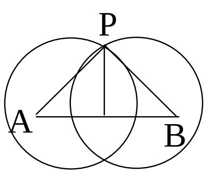
$\mathbf{x}=-2\pi,-\pi,0,\pi,2\pi$

86. If the angle of intersection at a point where the two circles with radii 5cm and 12cm intersect is 90°, then the length (in cm) of their common chord is:

1. $\frac{120}{13}$ **2.** $\frac{13}{2}$ **3.** $\frac{13}{5}$ **4.** $\frac{60}{13}$

Ans: 1

Sol:



Area $\Delta PAB=\frac{1}{2}dx=\frac{1}{2}r_1r_2$

$\frac{1}{2}.\sqrt{5^2+12^2}.x=\frac{1}{2}5\times 12$

$\Rightarrow x = \frac{5\times 12}{13}$

$\therefore 2x = \frac{120}{13}$

87. If A is a symmetric matrix and B is a skew symmetric matrix such that $A+B=\begin{bmatrix}2 & 3\\ 5 & -1\end{bmatrix}$, then AB is equal to:

1. $\begin{bmatrix}-4 & -2\\ -1 & 4\end{bmatrix}$ **2.** $\begin{bmatrix}4 & -2\\ 1 & -4\end{bmatrix}$ **3.** $\begin{bmatrix}4 & -2\\ -1 & -4\end{bmatrix}$ **4.** $\begin{bmatrix}-4 & 2\\ 1 & 4\end{bmatrix}$

Ans: 3

Sol: $A^T = A,\ B^T = -B$

$A+B=\begin{pmatrix}2 & 3\\ 5 & -1\end{pmatrix}$

$A = \frac{1}{2}\begin{bmatrix}2 & 3\\ 5 & -1\end{bmatrix} + \frac{1}{2}\begin{bmatrix}2 & 8\\ 3 & -1\end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix}4 & 8\\ 8 & -2\end{bmatrix} = \begin{bmatrix}2 & 4\\ 4 & -1\end{bmatrix}$

$B = \frac{1}{2}\begin{bmatrix}0 & -2\\ 2 & 0\end{bmatrix} = \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

$AB = \begin{bmatrix}2 & 4\\ 4 & -1\end{bmatrix} = \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

$AB = \begin{bmatrix}0+4 & -2+0\\ 0+(-1) & -4+0\end{bmatrix} = \begin{bmatrix}4 & -2\\ -1 & -4\end{bmatrix}$

88. If α and β are the roots of the equation $375x^2 - 25x - 2 = 0$, then

$\lim_{n\to\infty}\sum_{r=1}^{n}\alpha^r + \lim_{n\to\infty}\sum_{r=1}^{n}\beta^r$ **is equal to:**

1. $\frac{29}{358}$ **2.** $\frac{21}{346}$ **3.** $\frac{1}{12}$ **4.** $\frac{7}{116}$

Ans: 3

Sol: $\frac{\alpha}{1-\alpha} + \frac{\beta}{1-\beta} = \frac{\alpha - 2\beta + \beta - \alpha\beta}{(1-\alpha)(1-\beta)}$

$$=\frac{\alpha+\beta-2\alpha\beta}{1-(2+\beta)+\alpha\beta}=\frac{\frac{25}{375}-2\left(\frac{-2}{375}\right)}{1-\left(\frac{25}{375}\right)+\left(\frac{-2}{375}\right)}=\frac{25+4}{375-25-2}=\frac{29}{348}=\frac{1}{12}$$

89. If the truth value of the statement $p\to(\sim q\vee r)$ is false (F), then the truth values of the statements p, q, r are respectively:

1. T, F, T **2. T, F, F** **3. T, T, F** **4. F, T, T**

Ans: 3

Sol: $P\Rightarrow(\sim qVr)$

$T\Rightarrow(TVT)$

$T\Rightarrow(FVF)$ is F

90. Let P be the point of intersection of the common tangents to the parabola $y^2=12x$ and the hyperbola $8x^2-y^2=8$. If S and S' denote the foci of the hyperbola where S lies on the positive x – axis then P divides SS' in a ratio:

1. 13:11 **2. 14:13** **3. 2:1** **4. 5 : 4**

Ans: 4

Sol: $y=mx+\frac{3}{M}$ $\frac{x^2}{1}-\frac{y^2}{8}=1$ $\Rightarrow e^2=\frac{8+1}{1}=9$

$\frac{9}{m^2}=1m^2-8$ $9=m^4-8m^2$ $m=\pm 3$

Common tangents

$y=3x+1$

$y=-3x-1$

$2y=0$

$y=0$

$0=3x+1$

$x=\frac{-1}{3}$

point $P=\left(\frac{-1}{3},0\right)$ $S=(3,0),\ S'=(-3,0)$ $\frac{SP}{PS'}=\frac{3+\frac{1}{3}}{\frac{-1}{3}+3}=\frac{10}{8}=5/4$

✍... **Prepared by Sri Chaitanya Faculty**