Sri Chaitanya

IIT Academy., India

JEE - MAIN 2019

10th April 2019, Slot - 1

(9:30 am - 12:30 pm)

Question Paper

Solutions

Corporate Office : Plot No-304, Kasetty Heights, Ayyappa Society Madhapur, Hyderabad-500081

www.srichaitanya.net

PHYSICS

1. **A particle of mass m is moving along a trajectory given by** $x = x_0 + a\cos\omega_1 t$ $y = y_0 + b\sin\omega_2 t$ **The torque,acting on the particle about the origin, at t=0 is :**

1) $+my_0a\omega_1^2\,\hat{k}$ **2)** $m\left(-x_0b + y_0a\right)\omega_1^2\,\hat{k}$

3) $-m\left(x_0bw_2^2 - y_0aw_1^2\right)\hat{k}$ **4) Zero**

Ans. **1**

Sol. $\vec{r} = \left(x_o + a\cos\omega_1 t\right)i + \left(y_o + b\sin\omega_2 t\right)j$ **; at t=0** $\vec{r} = \left(x_o + a\right)i + y_o j$ $\vec{a} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j}$

$\vec{r} = \left(-a\omega_1^2\cos\omega_1 t\right)\hat{i} + \left(-b\omega_2^2\sin\omega_2 t\right)j$ **; at t=0** $\vec{a} = -a\omega_1^2\hat{i}$ $T = \vec{r}\times\vec{F} = m\left(\vec{r}\times\vec{a}\right) = m\left(y_o\left(-a\omega_1^2\right) - \hat{k}\right)$

$= my_o a\omega\hat{i}\hat{k}$

2. **A stationary source emits sound waves of frequency 500Hz. Two observers moving along a line passing through the source detect sound to be of frequencies 480 Hz ad 530Hz. Their respective speeds are,in ms^{-1}**
(given speed of sound =300 m/s)

1) 12, 18 **2) 8,18** **3) 16, 14** **4) 12,16**

Ans. **1**

Sol. $480 = 500\left(\frac{300 + V_o}{300}\right)\frac{24}{25} = 1 + \frac{V_o}{300}$ $\frac{V_o}{300} = \frac{1}{25}$ $|V_o| =$ **12** $530 = 500\left(\frac{300 + V_o}{300}\right)\frac{53}{50} = 1 + \frac{V_o}{300}$

$\frac{V_o}{300} = \frac{-3}{50}$ $|V_0| = 18$ **Hence Velocities -12,18**

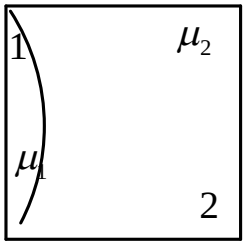
3. **n moles of an ideal gas with constant volume heat capacity C_v undergo an isobraric expansion by certain volume . The ratio of the work done in the process, to the heat supplied is :**

1) $\frac{4nR}{C_v + nR}$ **2)** $\frac{nR}{C_v + nR}$ **3)** $\frac{nR}{C_v - nR}$ **4)** $\frac{4nR}{C_v - nR}$

Ans. **2**

Sol. **n=moles** **cv=neat capacity** $\frac{P\left(v_2 - v_1\right)}{C_v\Delta t + P\left(v_2 - v_1\right)} = \frac{nR}{C_v - nR}$

4. **One Plano-convex and one plane of-concave lens of same radius of Curvature 'R' but of different materials are joined side by side as shown in the figure . If refractive index of the material of 1 is μ_1 and that of 2 is μ_2 ,then the focal length of the combination is**



1) $\frac{R}{2-(\mu_1-\mu_2)}$ 2) $\frac{2R}{\mu_1-\mu_2}$ 3) $\frac{R}{2(\mu_1-\mu_2)}$ 4) $\frac{R}{\mu_1-\mu_2}$

Ans. 4

Sol. Diagram $\frac{1}{F}=\frac{\mu_2-\mu_1}{-R}$ $f=\frac{R}{\mu_1-\mu_2}$

5. **In a photoelectric effect experiment the threshold wavelength of the is 380nm. If the wavelength of indent is 260nm, the maximum kinetic energy of emitted electrons will be: given E{in ev}=** $\frac{1237}{\lambda(innm)}$

1) 1.5 eV 2) 15.1 eV 3) 3.0 eV 4) 4.5 eV

Ans. 1

Sol. $N\frac{hc}{\lambda_o}=\frac{hc}{\lambda}+K.E\ \frac{hc}{260}=\frac{hc}{380}+K.E\ K.E=\frac{1240}{260}-\frac{1240}{380}=4.8-3.2$ **=1.5ev**

6. **The electric field of a plane electromagnetic wave is given by** $\vec{E}=E_0\hat{i}\cos(kz)\cos(\omega t)$ **The corresponding magnetic field** $\vec{B}$ *is then given by* :

1) $\vec{B}=\frac{E_0}{C}\hat{k}\sin(kz)\mathrm{Cos}(\omega t)$ 2) $\vec{B}=\frac{E_0}{C}\hat{j}\sin(kz)\cos(\omega t)$

3) $\vec{B}=\frac{E_0}{C}\hat{j}\sin(kz)\sin(\omega t)$ 4) $\vec{B}=\frac{E_0}{C}\hat{j}\cos(kz)\sin(\omega t)$

Ans. 3

Sol. $E=\frac{E_o}{2}\hat{i}\left[\cos(kz+\omega t)+\cos(kz-\omega t)\right]$ $B=\frac{B_o}{2}\hat{j}\left[\cos(kz-\omega t)-\cos(kz+\omega t)\right]$ $B=\frac{B_o}{2}\hat{j}\sin(kz)\sin\omega t$

7. **A transformer consisting of 300 turns in the primary and 150 turns in the secondary gives output power of 2.2kw. if the current in the secondary coil is 10A, then the input voltage and current in the primary Coil are:**

1) 440 V and 5A 2) 440 V and 20 A 3) 220 V and 10A 4) 220 V and 20A

1) 748J 2) 700J 3) 374J 4) 350J

Ans. 1

Sol. **N_p=300 N_s=150** $V_o=2.2kw=V_s.I_s\Rightarrow V_s=220V$ $\frac{N_P}{N_s}=\frac{V_p}{V_s}=\frac{I_s}{I_p}$ $I_p=\frac{V_sI_s}{Nlp}=5A$ $N_p=\frac{Np}{Ns}\times V_s=440v$

8. **A cylinder with fixed capacity of 67.2lit contains helium gas at STP. The amount of heat needed to raise the temperature of the gas by 20^0C is : [Given that R=8.31J** $mol^{-1}k^{-1}$ **]**

1) 748J 2) 700J 3) 374J 4) 350J

Ans. 1

Sol. **67.2 A7STP n= =nC$_V$** $\Delta 7$ $=3\times\frac{3R}{2}\times 20=9\times 3.14$ **=748**

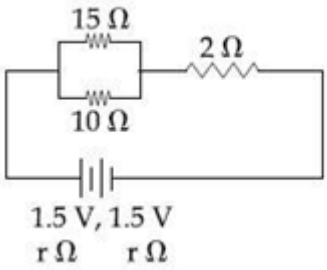
9. **Two radioactive materials A and B have decay constants 10λ and λ, respectively . If initially they have the same number of nuclei, then the ratio of the number of nuclei, then the ratio of the number of nuclei of A to that of B will be 1/e after a time**

1) $\frac{1}{9\lambda}$ **2)** $\frac{1}{11\lambda}$ **3)** $\frac{1}{10\lambda}$ **4)** $\frac{11}{10\lambda}$

Ans. **1**

Sol. $N_A = N_o e^{-10\lambda t}$ $N_B = N_o e^{-\lambda t}$ $\frac{N_A}{N_B} = e^{-9\lambda t} = \frac{1}{e}$ $9\lambda t = 1$ $t = \frac{1}{9\lambda}$

10. **In the given circuit,an ideal voltmeter connected across the 10 Ω resistance reads 2V. The internal resistance r, of each cell is**



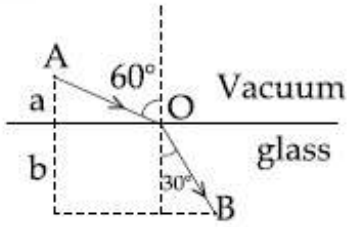
1) 0.5 Ω **2)** 0 Ω **3)** 1.5 Ω **4)** 1 Ω

Ans. **1**

Sol. $V_{AB} = 2\ 10i_1 = 2\ i_1 = \frac{1}{5}\ 15i_2 = 2\ i_2 = \frac{2}{15}\ i_1 + i_2 = \frac{1}{3}\ 3 - 2r(i_1 + i_2) - 10i_1 - 2(i_1 + i_2) = 0$

$1 - \frac{2}{3} = \frac{2r}{3}\ \frac{1}{3} = \frac{2r}{3}\ r = \frac{1}{2} = 0.5\Omega$

11. **A ray of light AO in vacuum is incident on a glass slab at angle 60^0 and refracted at angle 30^0 along OB as shown in the figure.The optical path length of light ray from A to B is:**



1) 2a+2b **2)** $\frac{2\sqrt{3}}{a} + 2b$ **3)** $2a + \frac{2b}{3}$ **4)** $2a + \frac{2b}{\sqrt{3}}$

Ans. **1**

Sol. **Optical length=AO+OB** $\frac{A}{90} = \sin 30^\circ = y_2$ $AO = 2a$ $\frac{b}{OB} = \cos 30 = \frac{\sqrt{3}}{2}$ $OB = \frac{2b}{\sqrt{3}}$

$\therefore$ **OA+OB=2a+** $\frac{2b}{\sqrt{3}}$

12. **A current of 5A passes through a copper conductor (resistivity=$1.7\times10^{-8}\Omega m$) radius of cross section 5mm.Find the mobility of the charges if their drift velocity is $1.1\times10^{-3}m/s.$**

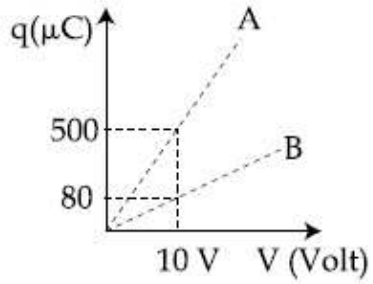
1) $1.8\times m^2/Vs.$ **2)** $1.5\times m^2/Vs.$ **3)** $1.0\times m^2/Vs.$ **4)** $1.3\times m^2/Vs.$

Ans. **3**

Sol. **i=neA(vd)** $j=\sigma E \quad n=\frac{i}{eAVd} \quad ne(vd)=\sigma E$ **Mobility=** $\frac{vd}{E} \quad \frac{i.e\,vd}{eAvd} p=E$

$$=\frac{(vd)A}{ip}=\frac{1.1\times10^{-3}\times\frac{22}{7}\times10^{-6}10^{-1}}{5\times1.7\times10^{-8}}=\frac{11}{17}\times\frac{22}{7}\times5\times10^{-1}=1m^2/V_s$$

13. **Figure shows charge (q) versus voltage (v) graph for series and parallel combination of two given capacitors . The capacitances are :**



1) $50\mu F \text{ and } 30\mu F$ **2)** $60\mu F \text{ and } 40\mu F$ **3)** $40\mu F \text{ and } 10\mu F$ **4)** $20\mu F \text{ and } 30\mu F$

Ans. **3**

Sol. **N**

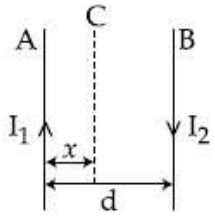
14. **The value of acceleration due to gravity at Earth's surface is 9.8ms^{-2} . The altitude above its surface at which the acceleration due to gravity decreases to 4.9 ms^{-2}, is close to (Radius of earth =** $6.4\times10^6 m.$**)**

1) $6.4\times10^6 m.$ **2)** $1.6\times10^6 m.$ **3)** $2.6\times10^6 m.$ **4)** $9.0\times10^6 m.$

Ans. **3**

Sol. $\frac{a}{c}=v\ 80=C_1+C_2\ 8=\frac{C_1C_2}{C_1+C_2}$ **400=(C_1 C_2)Options of (a) satisfy**

15. **Two wires A&B are carrying currents** $I_1 \& I_2$ **as shown in the figure . the separation between them is d . A third wire C carrying a current I is to be kept parallel to them at a distance x from A such that the net force acting on it is Zero . The possible values of x are:**



1) $x=\left(\frac{I_2}{I_1+I_2}\right)d$ and $x=\left(\frac{I_2}{I_1-I_2}\right)d$ 2) $x=\left(\frac{I_1}{I_1-I_2}\right)d$ and $x=\left(\frac{I_2}{I_1+I_2}\right)d$

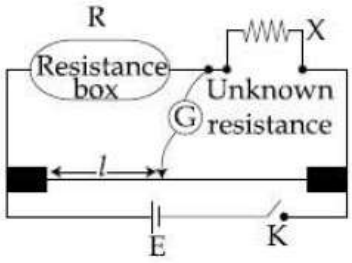
3) $x=\pm\frac{I_1 d}{(I_1-I_2)}$ 4) $x=\left(\frac{I_1}{I_1+I_2}\right)d$ and $x=\frac{I_2}{(I_1-I_2)}d$

Ans. 3

Sol. Diagram $\left(\frac{F}{l}\right)=\frac{\mu_0 I_1 I}{2\pi dx}=\frac{\mu_0 I_2 I}{2\pi(x-d)}$ $(x-d)I_1(x)I_2\ x(I_1-I_2)=dI_1$ $x=\frac{dI_1}{I_1-I_2}$ **If** $I_2>I_1$

Diagram $\left(\frac{F}{l}\right)=\frac{\mu_0 I_1 I}{(2\pi)x}=\frac{\mu_0 I_2 I}{2\pi(d+x)}$ $I_1(x+d)=(I_2)/x$ $x=\frac{I_1 d}{I_2-I_1}$ $\boxed{x=\frac{\pm Id_1}{I_1-I_2}}$

16. **In a meter bridge experiment the circuit diagram and the corresponding observation table are shown in figure .**



Sl. No.	R (Ω)	l (cm)
1.	1000	60
2.	100	13
3.	10	1.5
4.	1	1.0

Which of the readings is inconsistent ?

1) 4 **2) 3** **3) 1** **4) 2**

Ans. 1

Sol. We know that in meter bridge $\frac{R}{l}=\frac{x}{100-l}$ **a) R=1000, l=60** $\frac{1000}{60}=\frac{x}{40}\Rightarrow x=\frac{2}{3}\times 1000\simeq 666$

b) R=100, l=13 $\frac{100}{13}=\frac{x}{87}\Rightarrow x=\frac{87}{13}\times 100\simeq 666$ **c)** $\frac{10}{1.5}=\frac{x}{89.5}\Rightarrow x=\frac{89.5}{1.5}\times 10\simeq 666$

d) $\frac{1}{1}=\frac{x}{99}\Rightarrow x=99$ **Option 'd' is odd one So it is incontinent**

17. **An npn transistor operates as a common emitter amplifier , with a power gain of 60dB. The input circuit resistance is 100 Ω and the output load resitance is 10K Ω . The common emitter current gain β is :**

1) 10^4 **2) 60** **3)** 10^2 **4)** 6×10^2

Ans. 3

Sol. **60db=100Log** $\left(\frac{\beta R_L}{R_1}\right)$ $10^6=\frac{\beta R_L}{R_x}$ $10^6=\frac{\beta\times 10^4}{100}$ $\boxed{10^4=\beta}$

18. **A massage signal of frequency 100MHz and peak voltage 100V is used to execute amplitude modulation on a carrier wave of frequency 300GHz and peak voltage 400 V . The modulation index and difference between the two side band frequencies are**

1) $4;2\times 10^8\,Hz$ **2)** $0.25;2\times 10^8\,Hz$ **3)** $4;1\times 10^8\,Hz$ **4)** $0.25;1\times 10^8\,Hz$

Ans. 2

Sol. **Modulation index=** $\frac{100v}{400v}=0.25$ **Diff=** $2\omega_m=2\times10^7\,Hz$

19. **A uniformly charged ring of radius 3a and total charge q is placed in xy-plane centred at origin. A point charge q is moving towards the ring along the z-axis and has speed** $v\,at\,z=4a$ **The minimum value of v such that it crosses the origin is:**

1) $\sqrt{\frac{2}{m}\left(\frac{2}{15}\frac{q^2}{4\pi\in_0 a}\right)^{1/2}}$ **2)** $\sqrt{\frac{2}{m}\left(\frac{1}{15}\frac{q^2}{4\pi\in_0 a}\right)^{1/2}}$ **3)** $\sqrt{\frac{2}{m}\left(\frac{1}{5}\frac{q^2}{4\pi\in_0 a}\right)^{1/2}}$ **4)** $\sqrt{\frac{2}{m}\left(\frac{4}{15}\frac{q^2}{4\pi\in_0 a}\right)^{1/2}}$

Ans. 1

Sol. **Diagram** $E\perp$ **av to plane /ring=** $\frac{kqr}{\left(2^2+r^2\right)^{3/2}}$

$\therefore$ **Total workdone by electric field=change in KE** $\int_{4a}^{0}\frac{2kq^2\,2dn}{2\left(2^1+r^2\right)^{3/2}}=\frac{1}{2}mv4^2-\frac{1}{2}mv_1^{02}$

$$\int_{4a}^{0}\frac{kq^2dt}{2(t)^{3/2}}\frac{\frac{kq^2}{2}+\frac{-3}{2}+1}{\frac{-3}{2}+1}=\text{-}kq^2\frac{1}{ey_2}\int_{4a}^{o}=-kq^2\left[\frac{1}{a^2+3a}-\frac{1}{\sqrt{(x)-(3a)^2}}\right]-kq^2\left[\frac{1}{3a}-\frac{1}{5a}\right]$$

20. **A proton,an electron, and a Helium nucleus, have the same energy. they are in circular orbits in a plane due to magnetic field perpendicular to the plane. Let** r_p, r_e *and* r_{He} **be their respective radii, then,**

1) $r_e>r_p=r_{He}$ **2)** $r_e>r_p>r_{He}$ **3)** $r_e<r_p<r_{He}$ **4)** $r_e<r_p=r_{He}$

Ans. 4

Sol. $\left(\frac{mv}{qB}\right)\Rightarrow r\,\frac{1}{2}m_e v_e^2=k=\frac{1}{2}m_p v_p^2=\frac{1}{2}m_{me}\left(V_{He}\right)^2\;\therefore V_e\alpha\frac{1}{\sqrt{m}}\;\therefore V_e\alpha\frac{\sqrt{m}}{q}\;\therefore\gamma_{He}=(k')\frac{2}{(2)}=k'$

$\gamma_p=k'\left(\frac{1}{1}\right)=k'\;\gamma_e=k''<k'\;\therefore r_e<r_p=r_{He}$

21. **The ratio of surface tensions of mercury and water is given to be 7.5 while the ratio of their densities is 13.6. Their contact angles, with glass, are close to** 135^0 **and** 0^0 **respectively. it is observed that mercury gets depressed by an amount h in a capillary tube of radius** r_1 **, while water rises by the same amount h in a capillary tube of radius** r_2 **,the ratio,** (r_1/r_2) **is then close to:**

1) 4/5 **2) 2/5** **3) 2/3** **4) 3/5**

Ans. 2

Sol. $\frac{T_{Hg}}{T_{H_2O}}=7.5\frac{P_{Hg}}{P_{water}}=13.6$ **For a Capillar tube** $2\pi RT\cos\theta=\pi R^2hgP\;\frac{2T\cos\theta}{pgh}=R$

$$\begin{bmatrix}\theta Hg=1350 & \theta_{H_2O}=0^o\\ \theta_{Hg}=\frac{1}{\sqrt{2}} & \cos\theta H_2O=1\end{bmatrix}\;\frac{r_1}{r_2}\frac{2T_{Hg}Cos\theta Hg}{P_{Hg}gh}\frac{PH_zogh}{2T_{H_2O}Cos\theta H_2o}=\frac{T_{Hg}}{T_{H_2o}}\frac{P_{H_2O}}{P_{Hg}}\frac{Cos\theta_{Hg}}{Cos\theta_{H_2O}}=7.5\frac{1}{13.6}\times\frac{1}{\sqrt{2}}=\frac{2}{5}$$

22. **a moving Coil galvanometer allows a full scale current 10^{-4} A.A series resistance of2-$M\Omega$ is required to convert the above galvanometer into a voltmeter of range 0-5V. Therefore the value of shunt resistance required to convert the above galvanometer in to ammeter of range 0-10mA is :**

1) $100\,\Omega$ **2)** $10\,\Omega$ **3)** $500\,\Omega$ **4)** $200\,\Omega$

Ans. **4**

Sol. $10^{-4}(RG+RS)$ $V_{max} = i\,max/\,RG+RS$ $5 = 10^{-4}\left(RG+2\times10^{3}\right)$

$5\times10^{4} = R_G + 2\times10^{3}$ $48\times10^{3} = RG$ $I = \dfrac{i\,max(RG+RS)}{RS}$ $10\times10^{-3} = \dfrac{10^{-4}(RG+R)}{R}$

23. **A ball is thrown upward with an initial velocity V_0 from the surface of the earth. the motion of the ball is affected by a drag force equal to m γV^2 (where m is mass of the ball v is its instaneous velocity and γ is a constant) . Time taken by the ball to rise to its zenith is :**

1) $\dfrac{1}{\sqrt{\gamma g}}\sin^{-1}\left(\sqrt{\dfrac{\gamma}{g}}v_0\right)$ **2)** $\dfrac{1}{\sqrt{\gamma g}}\ln\left(1+\sqrt{\dfrac{\gamma}{g}}v_0\right)$

3) $\dfrac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\dfrac{\gamma}{g}}v_0\right)$ **4)** $\dfrac{1}{\sqrt{2\gamma g}}\tan^{-1}\left(\sqrt{\dfrac{2\gamma}{g}}v_0\right)$

Ans. **3**

Sol. **F_{net}ng-mrv^2=ma-(g+ev^2)=** $\dfrac{dv}{dt}$ $-gdt = \dfrac{dv}{\left(1+\left(\sqrt{\dfrac{r}{g}}\right)^2 v^2\right)}$ $\sqrt{\dfrac{r}{g}}$

$dv = \sqrt{\dfrac{g}{r}}\sec^2\theta dx$ $-gdt = \dfrac{d\theta}{\sqrt{\dfrac{r}{g}}\sec^2\theta}\sqrt{\dfrac{r}{g}}\sec^2\theta$ $\int_0^t dt = \int_0^{\theta}\dfrac{d\theta}{rg}$ $-t = \dfrac{1}{\sqrt{rg}}\tan - \sqrt{\dfrac{r}{g}}v\Big]_{v0}^{0}$

$+t = \dfrac{1}{\sqrt{rg}}0/\left(\tan\sqrt{\dfrac{r}{g}}v_o\right)$ $\therefore t = \dfrac{1}{\sqrt{rg}}\tan\left(\sqrt{\dfrac{r}{g}}v_o\right)$

24. **A thin disc of mass M and radius R has mass per unit area $\sigma(r)$ =kr^2 where r is the distance from its center . Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is :**

1) $\dfrac{MR^2}{3}$ **2)** $\dfrac{2MR^2}{3}$ **3)** $\dfrac{MR^2}{2}$ **4)** $\dfrac{MR^2}{6}$

Ans. **2**

Sol. $I = \sum dmr^2$ $I = \sum(2\pi x)dx\left(kx^2\right)x^2 = 2\pi k\int x^5dx = 2\pi k\left[\dfrac{x^6}{6}\right]_0^r = 2\pi k\dfrac{R^6}{6} = \dfrac{2\pi kR^6\times4}{6\times4} = \dfrac{2mR^2}{3}$

25. **Given below in the left column are different modes of communication using the kinds of waves given in the right column .**

A. Optical Fibre communication	**P. Ultra sound**
B. Radar	**Q. Infrared light**
C. Sonar	**R . Microwaves**
D. Mobile phones	**S. radiowaves**

From the options given below , find the most appropriate match between entries in the left and the right column.

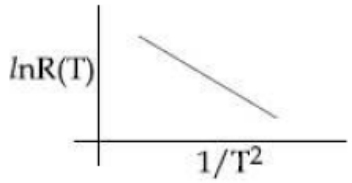
1) A-Q	**B-S,**	**C-P,**	**D-R**
2) A-S	**B-Q,**	**C-R,**	**D-P**
3) A-R	**B-P,**	**C-S,**	**D-Q**
4) A-Q	**B-S,**	**C-R,**	**D-P**

Ans. **1**

Sol. **Optical Fibre Communication: Infrared light Radar: Radio waves Sonar: Ultra Sound**

Mobile phones: micro waves Mode of communication

26. **In an experiment , the resistance of a material is plotted as a function of temperature (in some range) .As shown in the figure , It is a straight line**



one may conclude that:

1) $R(T)=R_0e^{T^2/T_0^2}$ **2)** $R(T)=R_0e^{-T_0^2/T^2}$ **3)** $R(T)=\frac{R_0}{T^2}$ **4)** $R(T)=R_0\,e^{-T^2/T_0^2}$

Ans. **2**

Sol. **In** $\frac{R(T)}{Rlo}=\frac{-K}{T^2}$ **R(T)=** $R_o e-k/T^2$

27. **A** $25\times10^{-3}\,m^3$ **volume cylinder is field with 1 mol O_2 gas room temperature (300k) The molicular diameter of O_2,and its root mean square speed , are found to be 0.3 nm and 200m/s, respectively.what is the average collision rate (per second) for an O_2 Molecule?**

1) $\sim10^{12}$ **2)** $\sim10^{13}$ **3)** $\sim10^{10}$ **4)** $\sim10^{11}$

Ans. **1**

Sol. **Molecular density=** $\frac{N_A}{25\times10^{-3}}$ **Diameter=0.3nm** V_{rms}**=200ms^{-1}** $\pi d^2\times dx\times n$

$\frac{\sqrt{2}nNA}{V}\pi d^2v=\text{collisionrate}$ **=$2.9\times10^9\approx10^{10}$**

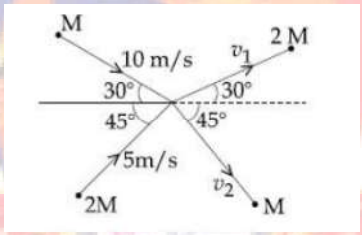
28. **Two Coaxial discs , having moments of inertia $I_1$and $\frac{I_1}{2}$ are rotating with respective angular velocity ω_1 and $\frac{\omega_1}{2}$ about their common axis.They are brought in contact with each other and thereafter they rotate with a common angular velocity . If E_f and E_i , are the final and initial total energies,then $\left(E_f - E_i\right)$ is**

1) $-\frac{I_1\omega_1^2}{12}$ **2)** $\frac{I_1\omega_1^2}{6}$ **3)** $-\frac{I_1\omega_1^2}{24}$ **4)** $\frac{3}{8}I_1\omega_1^2$

Ans. **3**

Sol. $\frac{1}{2}\left(\frac{I_1I_2}{I_1+I_2}\right)(\omega_{ra})^2$

29. **Two Particles , of masses M and 2M moving , as shown,with speeds of 10m/s and 5m/s , collied elastically at the origin. After the collision , they move along the indicated directions with speeds v_1 and v_2 respectively , The value of v_1 and v_2 are nearly:**



1) 3.2m/s and 12.6 m/s **2) 6.5m/s and 3.2 m/s**
3) 3.2m/s and 6.3 m/s **4) 6.5m/s and 6.3 m/s**

Ans. **4**

Sol. **X-axis** $(m)(10)\cos 30^\circ + (2m)(5)(\cos 45^\circ)(m)(v_2)\cos 45^\circ + (2m)(v_1)(\cos 30^\circ)$

$(10)\left(\frac{\sqrt{3}}{2}\right)+\frac{10}{\sqrt{2}}=\frac{V_2}{\sqrt{2}}+2v_1\frac{\sqrt{3}}{2}$ $5\sqrt{3}+\frac{10}{\sqrt{2}}=\frac{v_2}{\sqrt{2}}+v_1\sqrt{3}$ **.....(1)**

Y-Axis $-(m)(10)\sin 30^\circ + (2m)(5)\sin 45^\circ = m(-v_2)(\sin 45^\circ) + 2M(\sin 30^\circ)v_1$

$-\frac{10}{2}+\frac{10}{\sqrt{2}}=\frac{-v_2}{\sqrt{2}}+v_1$**.....(2) Solve 1 & 2 V_1=6.5m/s; V_2=6.3m/s**

30. **The displacement of a damped harmonic oscillator is given by** $x(t)=e^{-0.1t}\cos(10\pi t+\varphi)$ **Here t it is in seconds.The time taken for its amplitude of vibration to drop to half of its initial value is close to:**

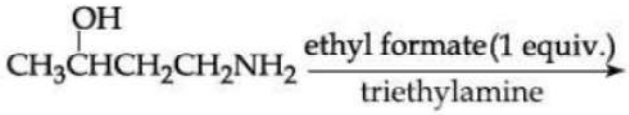
1) 27s **2) 13s** **3) 7s** **4) 4s**

Ans. **3**

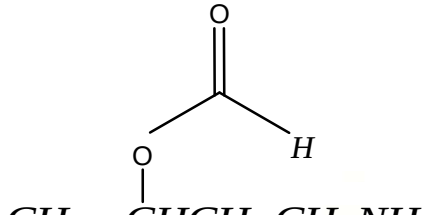
Sol. **Here A=Ae$^{-0.1t}$** $\frac{A_o}{2}=A_oe^{-0.1t}$ $0.1t=\ln 2=\frac{0.693}{0.1}=7s$

CHEMISTRY

31. The major product of the following reaction is



1) $CH_3CH=CH-CH_2NH_2$ **2)** $CH_3-\underset{OH}{\underset{|}{CH}}-CH=CH_2$

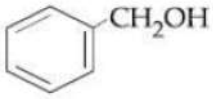
3)  **4)** $CH_3-\underset{OH}{\underset{|}{CH}}CH_2CH_2NHCHO$

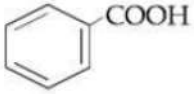
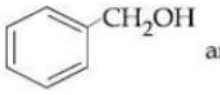
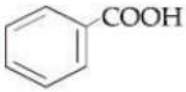
Ans. 4

Sol.

$$CH_3-\underset{OH}{\underset{|}{CH}}-CH_2-CH_2-NH_2+H-\underset{O}{\underset{||}{C}}-OH \rightarrow CH_3-\underset{OH}{\underset{|}{CH}}-CH_2-CH_2-NH-\underset{O}{\underset{||}{C}}-H$$

32. Major products of the following reaction are

1. HCOOH and  2. CH_3OH and HCO_2H

3. CH_3OH and  4.  and 

Ans. 1

Sol. **Aldehydes locking α – Hydrogen give cross cannizaro reaction.**

HCHO acts as hydride donor because it gives less sterically hindered tetrahedral Intermediat

33. Which of the following is a condensation polymer?

1) Buna-S 2) Teflon 3) Nylon 6,6 4) Neoprene

Ans. 3

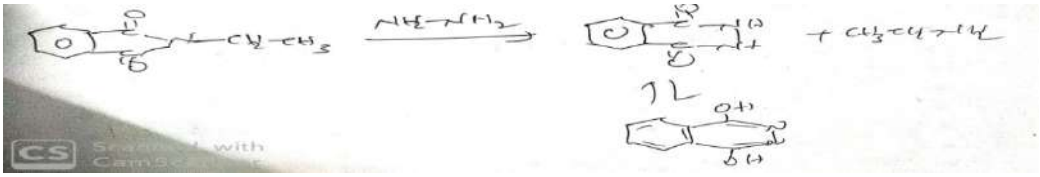
Sol. . Nylon 6,6,- is a conclusion polymer of $NH_2-(CH_2)_6-NH_2$ & Adinic Acid

34. Ethylamine $(C_2H_5NH_2)$ can be obtained from N-ethylphthalimide on treatment with

1) CaH_2 2) H_2O 3) $NaBH_4$ 4) NH_2NH_2

Ans. 4

Sol.



35. **A process will be spontaneous at all temperatures if :**

1) $\Delta H < 0 \, and \, \Delta S < 0$ **2)** $\Delta H < 0 \, and \, \Delta S > 0$

3) $\Delta H > 0 \, and \, \Delta S > 0$ **4)** $\Delta H > 0 \, and \, \Delta S < 0$

Ans. **2**

Sol.

$$\Delta G = \Delta H - T\Delta S$$

$$-ve \quad -ve \quad +ve$$

$$\downarrow \quad \downarrow$$

$$\Delta H < 0 \quad \Delta S > 0$$

36. **Amylopectin is composed of**

1) $\beta - D - glucose$ **C_1-C_4 and C_2-C_6 linkages**

2) $\beta - D - glucose$ **C_1-C_4 and C_1-C_6 linkages**

3) $\alpha - D - glucose$ **C_1-C_4 and C_1-C_6 linkages**

4) $\alpha - D - glucose$ **C_1-C_4 and C_2-C_6 linkages**

Ans. **3**

Sol. **Fact Based**

37. **At 300k and 1 atmospheric pressure 10ml of a hydrocarbon required 55mL of O_2 for complete combustion and 40mL of CO_2 is formed. The formula of the hydrocarbon is:**

1) C_4H_6 **2)** C_4H_7CI **3)** C_4H_8 **4)** C_4H_{10}

Ans. **1**

Sol. $Cxh_y + O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O \quad 10\left(x + \frac{y}{4}\right) = 55$

$$10x = 40 \Rightarrow x = 4 \Rightarrow x + \frac{y}{4} = 5.5 \Rightarrow \frac{y}{4} = 5.5 - 4 \Rightarrow \frac{y}{4} = 1.5 \Rightarrow y = 6$$

38. **A gas undergoes physical adsorption on a surface and follows the given freundlich adsorption isotherm equation $\frac{x}{m} = kp^{0.5}$ Adosrption of the gas increases with:**

1) increase in P and decrease in T **2) Decrease in P and decrease in T**

3) Increase in P and Increase in T **4) Decrease in P and Increase in T**

Ans. **1**

Sol. **. rate of physical adsorption** $\propto \frac{1}{Temp} \propto Pressure$

39. **The correct order of catenation is**

1) $Si > Sn > C > Ge$ **2)** $C > Sn > Si \approx Ge$ **3)** $Ge > Sn > Si > C$ **4)** $C > Si > Ge \approx Sn$

Ans. **4**

Sol. **Syn gas is used in production ($CO+H_2$) of methanol**

40. **The isoelectronic set of ions is :**

1) $Li^+, Na^+\ O^{2-}$ and F^- **2)** $N^{3-}, O^{2-}\ F^-$ and Na^+

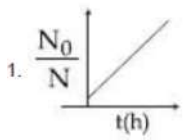
3) $F^-, Li^+\ Na^+$ and Mg^{2+} **4)** $N^{3-}, Li^+\ Mg^{2+}$ and O^{2-}

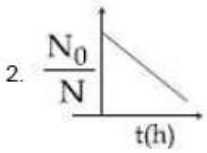
Ans. **2**

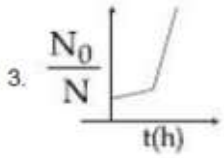
Sol. **All are having $10e^-$**

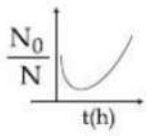
41. **A bacterial infection in an initial wound grows $N_0^1(t) = \exp(t)$, where the time t is in hours A dose of antibiotic, taken orally , needs 1 hour to reach the wound. Once it reaches there , the bacterial Population goes down as $\frac{dN}{dt} = -5N^2$**

What will be the plot of $\frac{N_0}{N}$ Vs t after 1 hour

1. 

2. 

3. 

4. 

Ans. **1**

Sol. **Initially, till t=1hr.**

$\frac{N_0}{N} = e^{-kt} - (J)\exp onentially\ \ decrea\sin g.$ **After t=1 Hr.**

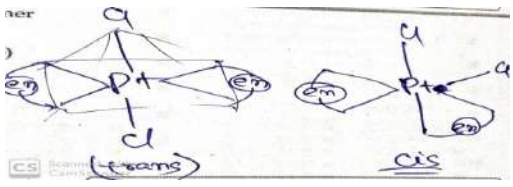
$$\frac{dN}{dt} = -5N^2 \int_{N_1}^{N} \frac{dN}{N^2} = -5\int_1^t dt \left[\frac{N^{-1}}{-1}\right]_{N_1}^{N} = -5t \Rightarrow \frac{1}{N} - \frac{1}{N_1} = 5t$$

$$Linear \Rightarrow \frac{N_1}{N} = 1 + 5 + N_1$$

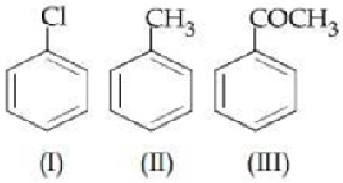
42. **The species that can have a trams-isomer (en=ethane-1,2-diamine,ox=oxalate)**

1) $[Cr(en)_2(ox)]^+$ **2)** $[Pr(en)_2Cl_2]^{2+}$ **3)** $[Zn(en)Cl_2]$ **4)** $[Pt(en)Cl_2]$

Ans. **2**

Sol. 

43. **The increasing order of the reactivity of the following compounds towards electrophilic aromatic substitution reactions is :**



1) III<I<II **2)III<II<I** **3) I<III<II** **4)II<I<III**

Ans. **1**

Sol. **Rate of electroplate aromatic substitution reaction depends on electron density in benzene ring. So it is increased by electron donating group.**

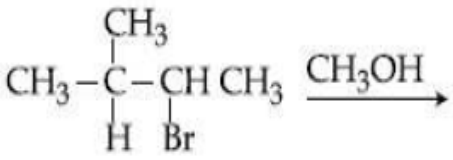
44. **During the change of O_2 to O_2^- ,the incoming electron goes to the orbital:**

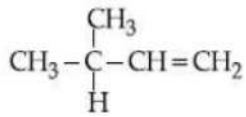
1) π^*2P_x **2)** $\pi 2P_x$ **3)** σ^*2P_z **4)** $\pi 2P_y$

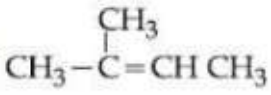
Ans. **1**

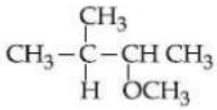
Sol. **For O_2 HOMO $\pi_{2_{Px}}$ is incompletely filled. So incoming C^- goes to $\pi_{2_{Px}}$**

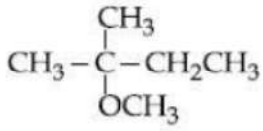
45. **The major product of the following reaction is :**



1. 

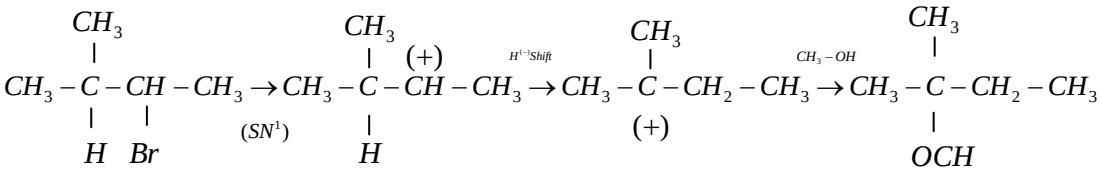
2. 

3. 

4. 

Ans. **4**

Sol.



46. **Consider the hydrated ions of Ti^{2+},V^{2+} ,Ti^{3+} and Sc^{3+} The correct order of their spin-only magnetic moments is :**

1. $Ti^{3+} < Ti^{2} < Sc^{3+} < v^{2+}$ **2)** $V^{2+} < Ti^{2+} < Ti^{3+} < Sc^{3+}$

3) $Sc^{3+} < Ti^{3+} < Ti^{2+} < V^{2+}$ **4)** $Sc^{3+} < Ti^{3+} < V^{2+} < Ti^{2+}$

Ans. **3**

Sol. $SC^{3+},3d^0$ $Ti^{2+},3d^2$ $Ti^{3+},3d^1$ $V^{2+},3d^3$

47. **Three complexes** $[CoCl(NH_3)_5]^{2+}(I)\,[Co(NH_3)_5H_2O]^{3+}(II)\,and\,[Co(NH_3)_6]^{3+}(III)$ **absorb light in the visible region . the correct order of the wavelength of light absorbed by them is :**

1) (II)>(I)>(III) 2) (III)>(I)>(II) 3) (I)>(II)>(III) 4) (III)>(II)>(I)

Ans. **3**

Sol. **More S.F.L, more is absorption, so Nabs ↓ frequency.**

III>II>I

III<II<I

48. **Consider the following table**

Gas	a/(kPa dm^6 mol^{-1})	b/(dm^3mol^{-1})
A.	642.32	0.05196
B.	155.21	0.04136
C.	431.91	0.05196
D.	155.21	0.4382

a and b are vander walls contants . The correct statement about the gases is :

1. Gas C will occupy lesser volume then gas A: gas B will be lesser Compressible than gas D

2) Gas C will occupy lesser volume than gas A: gas B will be more compressible than gas D

3) Gas C will occupy more volume than gas A: gas B will be more compressible than gas D

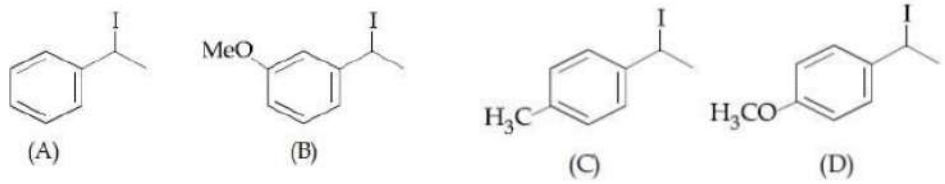
4) Gas C will occupy more volume than gas A: gas B will be lesser compressible than gas D

Ans. **3**

Sol. **Compressibility** $\propto a \propto \frac{1}{b}$

A is having higher intermolecular forces of attraction and hence occupies less volume.

49. **Increasing rate of** S_N1 **reaction in the following compounds is :**



(A) (B) (C) (D)

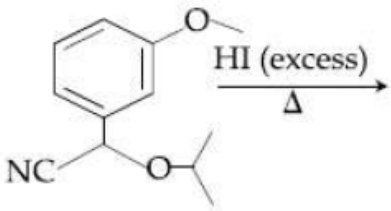
1) (B)<(A)<(C)<(D) **2) (A)<(B)<(D)<(C)**

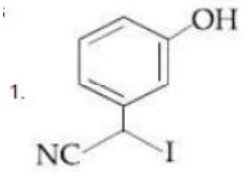
3) (B)<(A)<(D)<(C) **4) (A)<(B)<(C)<(D)**

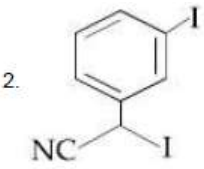
Ans. **1**

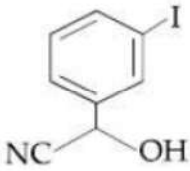
Sol. Rate of SN^1 reaction depends on stability of corbocation

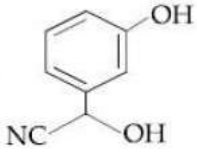
50. The major product of the following reaction is :



1. 

2. 

3. 

4. 

Ans. 4

Sol. in acidic medium Based on structure it may following SN^1 (or) SN^2 pathway.

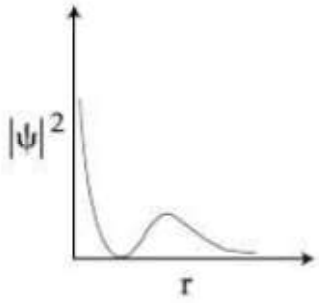
51. The alloy used in the contruction of aircrafts is :

1) Mg-A1 2) Mg-Mn 3) Mg-Zn 4)Mg-Sn

Ans. 1

Sol. Due alumin alloy (Mg-Al) is used in manufacturing of aircrafts

52. The graphbetween ψ^2 and r(radial diatance) is shown beow . This represents



1) 2s orbital 2) 2p orbital 3) 3s orbital 4 1s orbital

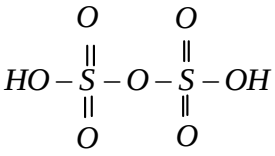
Ans. 1

Sol. No.of radial nodes=1 $\Rightarrow$ n-l-1=1 $\Rightarrow$ n-l=2 $\Rightarrow$ l=0 $\rightarrow$ n=2 $\Rightarrow$ 2s l=1 $\rightarrow$ n=3 $\Rightarrow$ 3p

53. The oxoacid of sulphur that does not contain bond between sulphur atoms is :

1) $H_2S_2O_3$ 2) $H_2S_2O_7$ 3) $H_2S_4O_6$ 4) $H_2S_2O_4$

Ans. 2



Sol. No S-S bond

54. The principle of Column Chromatography is

1) Capillary action

2) Differential absorption of the substances on the solid phase

3) Differential absorption of the substances on the phase

4) Gravation forc

Ans. 3

Sol. Column Chromatography is based on difference in adsorption tendany of different substances in stationary phase.

55. At room temperature a dilute solution of urea is prepared by dissolving 0.60g of urea in 360g of water . If the vapour pressure of pure water at this temperature is 35mmHg , Lowering of vapour pressure will be (molar mass of urea =60g mol^{-1})

1) 0.028mmHg 2) 0.031 mmHg 3) 0.027Hg 4) 0.017mmHg

Ans. 4

Sol. $x_{urea} = \frac{0.6}{60} = 0.01 \quad x_{H_2O} = \frac{360}{18} = 20 \quad x_{urea} = \frac{0.01}{20+0.1} = 0.5\times10^{-3}$

$\therefore \Delta_P = xp^O_{H_2O} = 0.5\times10^{-3} = 0.175 mmofhg$

56. The synonym for water gas when used in the production of methanol is :

1) Laughing gas 2) syngas 3) fuel gas 4) natural gas

Ans. 2

Sol. Syn gas is used in production ($CO+H_2$) of methanol.

57. Consider the following statements

a) The pH of a mixture containing 400mL of 0.1M H_2SO_4 and 400 mLof 0.1M NaOH will be approximately 1.3

b) Ionic product of water is temperature dependent

C) A monobasic acid with $K_a=10^{-5}$ has a pH=5. The degree of dissociation of this acid is 50%

d) The Le Chatelier's Principle is not applicable to common-ion effect

The correct statements are

1) b and c 2) a,b and d 3) a,b, and c 4) a and b

Ans. 3

Sol.

$$H_2SO_4 + NaOH$$
$$\downarrow \quad + \quad \downarrow$$
$$H^+ \quad + \quad OH^- \rightarrow H_2O$$

I=400x0.1 **400x0.1**

=80meq **=40meq**

F= 40meq **0** $\therefore \left[H^+\right] = \frac{40}{800} = \frac{1}{20}$ $pH = -\log\frac{1}{20} = \log_2 + \log_{10} = 13$

58. **Consider the statement S1 snd S2:**

S1: Conductivity always increases with decreases in the concentraction of electrolyte

S2: Molar Conductivity always increase with decreases in the concentration of Electrolyte The correct option amoung the following

1) S1 is correct and S2 is wrong **2) both S1 and S2 are correct**

3) Both S1 and s2 are wrong **4) S1 is wrong S2 is correct**

Ans. 4

Sol. **Conductivity always ↑ with ↑ in conc. of ionc.**

Molar conductivity ↑ as dilution ↑

59. **Match the refining methods (column I) Watch metals (column-II)**

Column_I (Refinging methods)	**Clumn_II (Metals)**
I) Liquation	**a) Zr**
II) Zone refinging	**b) Ni**
III) Mond process	**c) Sn**
iV) Van Arkel Method	**d) Ga**

1) I-b	**II- c**	**III-d**	**IV-a**
2) I-c	**II-d**	**III-b**	**IV-a**
3) I-c	**II-a**	**III-b**	**IV-d**
4) I-b	**II-d**	**III-a**	**IV-c**

Ans. 2

Sol. **Van arkel- Zr Mond's process – Ni Zone refining- Ga Liquation-Sn**

60. **The regions of the atmosphere , Where clouds form where we live repectively, are**

1) Troposphere and stratosphere **2) Stratosphere and Troposphere**

3) Stratosphere and Stratosphere **4) Troposphere and Troposphere**

Ans. 4

Sol. **Troposphere 6-10 km from sea lavel Stratosphere =10-50km.**

MATHEMATICS

61. **The number of 6 digits numbers that can be formed using the digits 0,1,2,5,7 and 9 which are divisible by 11 and no digit is repeated, is:**

1) 36 **2) 48** **3) 60** **4) 72**

Ans. **3**

Sol. **apbqcr is 6 digit number is divisible by 11**

If $|(a+b+c)-(p+q+r)|$ **is 0 or multiple of 11**

There is only one possible way $|(9+2+1)-(7+5+0)|=0$

$a,b,c\in\{9,2,1\}$ *and* $p,q,r\in\{7,5,0\}$ **or** $a,b,c\in\{7,5,0\}$ *and* $p,q,r\in\{9,2,1\}$

Number of 6 digits numbers = $3!.3!+2.2!.3!=36+24=60$

62. **If** $\Delta_1=\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ *and* $\begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}, x\neq 0;$

then for all $\theta\in\left(0,\frac{\pi}{2}\right)$:

1) $\Delta_1-\Delta_2=x(\cos 2\theta-\cos 4\theta)$ **2)** $\Delta_1+\Delta_2=-2x^3$

3) $\Delta_1+\Delta_2=-2(x^3+x-1)$ **4)** $\Delta_1-\Delta_2=-2x^3$**.**

Ans. **2**

Sol. $\Delta_1=-x^3=\Delta_2$

63. **If** $a_1,a_2,.....a_n$ **are in A.P. and** $a_1+a_4+a_7+a_{10}+a_{13}+a_{16}=114,$ **then** $a_1+a_6+a_{11}+a_{16}$ **is equal to**

1) 38 **2) 64** **3) 76** **4) 98**

Ans. **3**

Sol. **Given** $6a+45d=114\Rightarrow 2a+15d=38$ **We need** $4a+30d=76$

64. **If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression** $(1+ax+bx^2)(1-3x)^{15}$**, in powers of x, then the ordered pair (a, b) is equal to**

1) (28, 315) **2) (-54, 315)** **3) (28, 861)** **4) (-21, 714)**

Ans. **1**

Sol. $(1+ax+bx^2)(1-3x)^{15}=(1+ax+bx^2)\left(1-{}^{15}C_1(3x)+{}^{15}C_2(3x)^2-{}^{15}C_3(3x)^3+....\right)$

Coefficient of x^2 **is** ${}^{15}C_2 3^2-a\cdot{}^{15}C_1\cdot 3+b=0\Rightarrow 45a-b=945$

Coefficient of x^3 **is** $-{}^{15}C_3\cdot 3^3+a\cdot{}^{15}C_2\cdot 3^2-b\cdot{}^{15}C_1\cdot 3=0\Rightarrow 21a-b=273$

Solving above equations, we get $a=28,b=315$

65. If $a>0$ and $z=\frac{(1+i)^2}{a-i}$, and $|z|=\sqrt{\frac{2}{5}}$ then $\bar{z}$ is equal to

1) $-\frac{1}{5}+\frac{3i}{5}$ **2)** $\frac{1}{5}-\frac{3i}{5}$ **3)** $-\frac{1}{5}-\frac{3i}{5}$ **4)** $-\frac{3}{5}-\frac{i}{5}$

Ans. 3

Sol. $z=\frac{2i}{a-i}$ and $|z|=\sqrt{\frac{2}{5}}\Rightarrow\frac{2}{\sqrt{a^2+1}}=\sqrt{\frac{2}{5}}\Rightarrow a=3(\because a>0)\ z=\frac{2i}{3-i}=-\frac{1}{5}+\frac{3}{5}i\Rightarrow\bar{z}=-\frac{1}{5}-\frac{3}{5}i$

66. If $\lim_{x\to1}\frac{x^4-1}{x-1}=\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$; then k is

1) $\frac{3}{8}$ **2)** $\frac{3}{2}$ **3)** $\frac{4}{3}$ **4)** $\frac{8}{3}$

Ans. 4

Sol. $\lim_{x\to1}\frac{x^4-1}{x-1}=4$ and $\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}=\lim_{x\to k}\frac{\frac{x^3-k^3}{x-k}}{\frac{x^2-k^2}{x-k}}=\frac{3}{2}k$ So $\frac{3}{2}k=4\Rightarrow k=\frac{8}{3}$

67. $\lim_{n\to\infty}\left(\frac{(n+1)^{1/3}+(n+2)^{1/3}+.....+(2n)^{1/3}}{n^{4/3}}\right)$ is equal to

1) $\frac{4}{3}(2)^{4/3}$ **2)** $\frac{3}{4}(2)^{4/3}-\frac{3}{4}$ **3)** $\frac{3}{4}(2)^{4/3}-\frac{4}{3}$ **4)** $\frac{4}{3}(2)^{3/4}$

Ans. 2

Sol. Given sum= $\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\left(1+\frac{r}{n}\right)^{1/3}=\int_0^1(1+x)^{1/3}dx=\frac{3}{4}\left(2^{4/3}-1\right)$

68. If y=y(x) is the solution of the differential equation $\frac{dy}{dx}=(\tan x-y)\sec^2 x$, $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, such that $y(0)=0$, then $y\left(-\frac{\pi}{4}\right)$ is equal to

1) $2+\frac{1}{e}$ **2)** $e-2$ **3** $\frac{1}{2}-e$ **4)** $\frac{1}{e}-2$

Ans. 2

Sol. Put $\tan x=t\Rightarrow\frac{dy}{dt}=t-y\Rightarrow-\frac{d}{dt}(t-y)=t-y-1\Rightarrow\frac{d(t-y)}{t-y-1}=-dt\Rightarrow\ln|t-y-1|=-t+C$

So, $\tan x+\ln|\tan x-y-1|=C\quad y(0)=0\Rightarrow C=0$

So the equation will be $\tan x+\ln(-\tan x+y+1)=0$ Put $x=-\frac{\pi}{4}\Rightarrow y+2=e\Rightarrow y=e-2$

69. If α and β are roots of the equation $x^2+\sin\theta\, x-2\sin\theta=0, \theta\in\left(0,\frac{\pi}{2}\right)$, then $\frac{\alpha^{12}+\beta^{12}}{\left(\alpha^{-12}+\beta^{-12}\right)(\alpha-\beta)^{24}}$ is equal to

1) $\frac{2^6}{(8+\sin\theta)^{12}}$ 2) $\frac{2^{12}}{(8+\sin\theta)^{12}}$ 3) $\frac{2^{12}}{(8-\sin\theta)^{6}}$ 4) $\frac{2^{12}}{(4-\sin\theta)^{12}}$

Ans. 2

Sol. $\alpha+\beta=-\sin\theta,\ \alpha\beta=-2\sin\theta \Rightarrow (\alpha-\beta)^2=\sin^2\theta+8\sin\theta$

$$\frac{\alpha^{12}+\beta^{12}}{\left(\alpha^{-12}+\beta^{-12}\right)(\alpha-\beta)^{24}}=\frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}=\frac{2^{12}}{(\sin\theta+8)^{12}}$$

70. If the length of perpendicular from point $(\beta,0,\beta)$, (where $\beta\neq0$) to the line $\frac{x}{1}=\frac{y-1}{0}=\frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then value of β is

1) -1 2) -2 3) 1 4) 2.

Ans. 1

Sol. Let $P(t,1,-1-t)$ be the foot of perpendicular from $A(\beta,0,\beta)$ on $\frac{x}{1}=\frac{y-1}{0}=\frac{z+1}{-1}$

AP having d.r.s $<t-\beta,1,-1-t-\beta>$ should be perpendicular to line with d.r.s $<1,0,-1>$

$\Rightarrow t-\beta+1+t+\beta=0 \Rightarrow t=-\frac{1}{2}$ Perpendicular distance = $\sqrt{(t-\beta)^2+1^2+(-1-t-\beta)^2}=\sqrt{\frac{3}{2}}$

$\Rightarrow 2\left(\beta+\frac{1}{2}\right)^2+1=\frac{3}{2}\Rightarrow\beta+\frac{1}{2}=\pm\frac{1}{2}\Rightarrow\beta=0,-1$ Since $\beta\neq0,$ we have $\beta=-1$

71. The line x =y touches a circle at the point P (1, 1). If the circle also passes through the point (1,-3), then its radius :

1) 2 2) $2\sqrt{2}$ 3) 3 4) $3\sqrt{2}$

Ans. 2

Sol. The required circle will be $(x-1)^2+(y-1)^2+\lambda(x-y)=0$

It passes through $(1,-3)\Rightarrow(1-1)^2+(-3-1)^2+\lambda(1+3)=0\Rightarrow\lambda=-4$

So equation of circle will be $x^2+y^2-6x+2y+2=0$ having radius as $2\sqrt{2}$

72. If the circles $x^2+y^2+5kx+2y+k=0$ and $x^2+y^2+kx+\frac{3y}{2}-\frac{1}{2}=0,(k\in R)$ Intersect at the points P and Q, then the line $4x+5y-k=0$ passés through P and Q, for:

1) no value of k 2) exactly 2 values of k
3) infinitely many values of k 4) exactly one value of k

Ans. 2

Sol. Chord PQ is radical axis which is nothing but $S-S'=0 \Rightarrow 4kx+\frac{1}{2}y+k+\frac{1}{2}=0$

Comparing it with $4x+5y-k=0$, we get $\frac{4k}{4}=\frac{1/2}{5}=\frac{k+1/2}{-k}$

On solving we get 2 different values of *k*

73. The region represented by $|x-y|\le 2$ *and* $|x+y|\le 2$ is bounded by a :

1) rhombus of side length 2 units
2) Square of area is 16 sq. units
3) rhombus of area $8\sqrt{2}$ sq. units
4) Square of side length $2\sqrt{2}$ un

Ans. 4

Sol. When we plot the graph, we get the region bounded by $\pm x\pm y=2$, which is a square of side $2\sqrt{2}$

74. If $f(x)=\begin{cases}\frac{\sin(p+1)x+\sin x}{x}, & x<0\\ q, & x=0\\ \frac{\sqrt{x^2+x}-\sqrt{x}}{x^{3/2}}, & x>0\end{cases}$ is continuous at x = 0, then the ordered pair (p, q) is equal to:

1) $\left(-\frac{1}{2},\frac{3}{2}\right)$
2) $\left(-\frac{3}{2},-\frac{1}{2}\right)$
3) $\left(\frac{5}{2},\frac{1}{2}\right)$
4) $\left(-\frac{3}{2},\frac{1}{2}\right)$

Ans. 4

Sol. $f(x)$ is continuous at $x=0 \Rightarrow f(0^+)=f(0)=f(0^-)$

$$\Rightarrow \lim_{x\to 0^-}\frac{\sin(p+1)x+\sin x}{x}=q=\lim_{x\to 0^+}\frac{\sqrt{x^2+x}-\sqrt{x}}{x^{3/2}}$$

$$\Rightarrow p+1+1=q=\lim_{x\to 0^+}\frac{\sqrt{x+1}-1}{x} \Rightarrow p+2=q=\lim_{x\to 0^+}\frac{1+\frac{x}{2}-1}{x} \Rightarrow p+2=q=\frac{1}{2} \therefore (p,q)=\left(-\frac{3}{2},\frac{1}{2}\right)$$

75. Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls is :

1) $\frac{1}{17}$
2) $\frac{1}{10}$
3) $\frac{1}{11}$
4) $\frac{1}{12}$

Ans. 3

Sol. At least 2 girls is mentioned in question

	Family1	Family2
2 BOYS AND 2 GIRLS	BB	GG
	BG	BG
	BG	GB
	GB	GB
	GB	BG
	GG	BB
1 BOY AND 3 GIRLS	BG	GG
	GB	GG
	GG	BG
	GG	GB
4 GIRLS	GG	GG

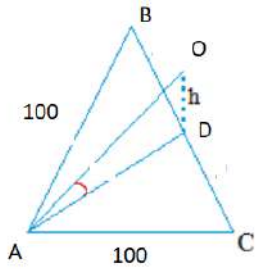
P(getting 4 girls) =1/11

76. ABC is a triangular park with AB = AC = 100 m. A vertical tower is situated at the midpoint of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}\left(3\sqrt{2}\right)$ and $\operatorname{cosec}^{-1}\left(2\sqrt{2}\right)$ respectively, then the height of the tower (in metres) is:

1) 25 **2)** $10\sqrt{5}$ **3)** $\frac{100}{3\sqrt{3}}$ **4) 20**

Ans. 4

Sol.



Given $\frac{OD}{AB}=\frac{1}{3\sqrt{2}}$ **and** $\frac{OB}{OD}=2\sqrt{2}$

$(OB)^2=(OD)^2+(BD)^2$ **and** $100^2=(AD)^2+(BD)^2$ $\therefore$ **OD=20**

77. If $\int\frac{dx}{\left(x^2-2x+10\right)^2}=A\left(\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{f(x)}{x^2-2x+10}\right)+C$ **where C is a constant of integration, then**

1) $A=\frac{1}{54}, f(x)=3(x-1)$ **2)** $A=\frac{1}{54}, f(x)=9(x-1)^2$

3) $A=\frac{1}{27}, f(x)=9(x-1)$ **4)** $A=\frac{1}{81}, f(x)=3(x-1)$

Ans. 1

Sol. Put $x-1=t\Rightarrow I=\int\frac{dt}{\left(t^2+9\right)^2}=\int\frac{\frac{1}{t^2}}{t^2+\frac{81}{t^2}+18}dt=\frac{1}{18}\int\frac{1+\frac{9}{t^2}}{\left(t-\frac{9}{t}\right)^2+36}dt-\frac{1}{18}\int\frac{1-\frac{9}{t^2}}{\left(t+\frac{9}{t}\right)^2}dt$

$$\Rightarrow I=\frac{1}{54}\tan^{-1}\left(\frac{t}{3}\right)+\frac{1}{18}\left(\frac{t}{t^2+9}\right)+C$$

78. **If the line $x-2y=12$ is tangent to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $\left(3,-\frac{9}{2}\right)$, then the length of latus rectum of ellipse is**

1) 9 **2)** $12\sqrt{2}$ **3)** $8\sqrt{3}$ **4)** 5

Ans. 1

Sol. $\left(3,-\frac{9}{2}\right)$ **lie on** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\Rightarrow\frac{9}{a^2}+\frac{81}{4b^2}=1$ $y=\frac{1}{2}x-6$ **is tangent** $\Rightarrow\sqrt{\frac{a^2}{4}+b^2}=6$

On solving we get length of latus rectum as $\frac{2b^2}{a}=9$

79. **If for some $x\in R$, the frequency distribution of the marks obtained by 20 students in a test is:**

Marks	2	3	5	7
No. of students	$(x+1)^2$	$2x-5$	x^2-3x	x

Then the mean of the marks is:

1) 2.5 **2)** 3.0 **3)** 2.8 **4)** 3.2

Ans. 3

Sol. $(x+1)^2+(2x-5)+(x^2-3x)+x=20\Rightarrow x^2+x-12=0\Rightarrow x=3$

Mean= $\frac{\sum_{i=1}^{4}n_ix_i}{\sum_{i=1}^{4}n_i}=\frac{2(x+1)^2+3(2x-5)+5(x^2-3x)+7x}{20}=2.8$

80. **Let $f(x)=e^x-x$ and $g(x)=x^2-x,\forall x\in R$. Then the set of all $x\in R$, where the function h(x)=(fog)(x) is increasing, is:**

1) $\left[-\frac{1}{2},0\right]\cup[1,\infty)$ **2)** $\left[-1,-\frac{1}{2}\right]\cup\left[\frac{1}{2},\infty\right)$ **3)** $[0,\infty)$ **4)** $\left[0,\frac{1}{2}\right]\cup[1,\infty)$

Ans. 4

Sol. $h'(x)\geq 0\Rightarrow f'(g(x))g'(x)\geq 0\Rightarrow\left(e^{x^2-x}-1\right)(2x-1)\geq 0\Rightarrow(x^2-x)(2x-1)\geq 0$

81. **If a directrix of a hyperbola centred at origin and passing through $\left(4,-2\sqrt{3}\right)$ is $5x=4\sqrt{5}$ and its eccentricity of hyperbola (e) satisfy the equation**

1) $4e^4-24e^2+27=0$ **2)** $4e^4+8e^2-35=0$

3) $4e^4-24e^2+35=0$ **4)** $4e^4-12e^2-35=0$.

Ans. 3

Sol. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1,\ x=\frac{a}{e}=\frac{4}{\sqrt{5}}$ $a=\frac{4e}{\sqrt{5}},b^2=a^2(e^2-1)$ $\left(4,-2\sqrt{3}\right)$ **lie on** $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$$\Rightarrow \frac{16}{a^2}-\frac{12}{b^2}=1 \Rightarrow \frac{16}{a^2}-\frac{12}{a^2\left(e^2-1\right)}=1 \Rightarrow \frac{16}{\left(\frac{4e}{\sqrt{5}}\right)^2}-\frac{12}{\left(\frac{4e}{\sqrt{5}}\right)^2\left(e^2-1\right)}=1 \Rightarrow 4e^4-24e^2+35=0$$

82. **Let** $S=\frac{3\times 1^3}{1^2}+\frac{5\left(1^3+2^3\right)}{1^2+2^2}+\frac{7\left(1^3+2^3+3^3\right)}{1^2+2^2+3^2}+\ldots\ldots$ **then upto 10th terms is**

1) 620 **2) 600** **3) 680** **4) 660**

Ans. **4**

Sol. $S=\sum_{n=1}^{10}(2n+1)\frac{6n^2(n+1)^2}{4n(n+1)(2n+1)}=\sum_{n=1}^{10}\frac{3n(n+1)}{2}$ **=660**

83. **Let** $f(x)=x^2, x\in R$. **For any** $A\subseteq R$, **define** $g(A)=\{x\in R: f(x)\in A\}$.

If $S=[0,4]$, **then which one of the following statements is not true?**

1) $f(g(S))\neq f(S)$ **2)** $f(g(S))=S$

3) $g(f(S))=g(S)$ **4)** $g(f(S))\neq S$.

Ans. **3**

Sol. $f(S)=[0,16]$ **and** $g(S)=[-2,2]$ **and** $g(f(S))=[-4,4]$ **and** $f(g(S))=[0,4]$

84. **Which one of the following Boolean expressions is a tautology?**

1) $(p\vee q)\vee(p\vee\sim q)$ **2)** $(p\wedge q)\vee(p\wedge\sim q)$

(iii) $(p\vee q)\wedge(p\vee\sim q)$ **(iv)** $(p\vee q)\wedge(\sim p\vee\sim q)$

Ans. **1**

Sol. **Take** $p=F\ \&\ q=T$, **we get** $(p\vee q)\wedge(p\vee\sim q)$ **and** $(p\wedge q)\vee(p\wedge\sim q)$ **as F**

Take $p=F\ \&\ q=F$, **we get** $(p\vee q)\wedge(\sim p\vee\sim q)$ **as F**

85. **The value of** $\int_0^{2\pi}\left[\sin 2x(1+\cos 3x)\right]dx$, **(where [t] denotes Greatest Integer Function)**

1) -2π **2)** π **3)** 2π **4)** $-\pi$

Ans. **4**

Sol. $I=\int_0^{2\pi}\left[\sin 2x(1+\cos 3x)\right]dx$ **using f(a + b - x) property**

$$I=\int_0^{2\pi}\left[-\sin 2x(1+\cos 3x)\right]dx \ \therefore I+I=\int_0^{2\pi}(-1)\,dx=-x\Big]_0^{2\pi} \Rightarrow I=-\pi$$

86. **If the system of linear equations** $x+y+z=5, x+2y+2z=6, x+3y+\lambda z=\mu,$ $(\lambda,\mu\in R)$ **has infinitely many solutions, then the value of** $\lambda+\mu$ **is**

1) 7 **2) 10** **3) 12** **4) 9**

Ans. **2**

Sol. $[AD]=\begin{bmatrix}1&1&1&5\\1&2&2&6\\1&3&\lambda&\mu\end{bmatrix}\sim\begin{bmatrix}1&1&1&5\\0&1&1&1\\0&2&\lambda-1&\mu-5\end{bmatrix}R_2-R_1, R_3-R_1$

$\sim\begin{bmatrix}1&1&1&5\\0&1&1&1\\0&0&\lambda-3&\mu-7\end{bmatrix}R_3-2R_2\ \infty$ **many solutions if** $\lambda-3$**=0,** $\mu-7$**=0** $\Rightarrow\lambda+\mu=10$

87. **Let** $A(3,0,-1), B(2,10,6)$ *and* $C(1,2,1)$ **be the vertices of a triangle and M is the midpoint of AC. If G divides BM in the ratio, 2 :1, then** $\cos(\angle GOA)$ **(O being origin) is equal to :**

1) $\frac{1}{2\sqrt{15}}$ **2)** $\frac{1}{\sqrt{15}}$ **3)** $\frac{1}{6\sqrt{10}}$ **4)** $\frac{1}{\sqrt{30}}$

Ans. **2**

Sol. **BM is median and G is centroid of the triangle G(2, 4, 2) and A(3, 0, -1)**

dr's of OG = 1,2,1 and dr's of OA = 3, 0, -1

$\cos\angle GOA=\frac{3+0-1}{\sqrt{1+4+1}\sqrt{9+0+1}}=\frac{1}{\sqrt{15}}$

88. **If** $Q(0,-1,-3)$ **is the image of P in the plane** $3x-y+4z-2=0$ **and R is the point** $(3,-1,-2)$**. then the area (in sq.units) of** ΔPQR **is :**

1) $\frac{\sqrt{91}}{4}$ **2)** $\frac{\sqrt{91}}{2}$ **3)** $2\sqrt{13}$ **4)** $\frac{\sqrt{65}}{2}$.

Ans. **2**

Sol. $\frac{x_1-0}{3}=\frac{y_1+1}{-1}=\frac{z_1+3}{4}=\frac{-2(1-12-2}{9+1+16}\Rightarrow P(x_1,y_1,z_1)=(3,-2,1)\ Q=(0,-1,3)$ *and* $R=(3,-1,-2)$

Area of $\Delta PQR = \frac{1}{2}\left|\overline{PQ}\times\overline{PR}\right| = \frac{1}{2}\sqrt{91}$ **sq.units**

89. **All the pairs (x,y) that satisfy the inequality** $\frac{2^{\sqrt{\sin^2 x-2\sin x+5}}}{4^{\sin^2 y}} \le 1$ **also satisfy the equation:**

1) $2\sin x = \sin y$ **2)** $\sin x = 2\sin y$ **3)** $\sin x = |\sin y|$ **4)** $2|\sin x| = 3\sin y$

Ans. **3**

Sol. $2^{\sqrt{\sin^2 x-2\sin x+5}} \le 4^{\sin^2 y} \Rightarrow 2^{\sqrt{(\sin x-1)^2+4}} \le 4^{\sin^2 y}$

LHS min value is 4 and RHS max value is 4

$\Rightarrow$ **LHS=RHS=4** $\Rightarrow$ **sin x = 1 and sin^2 y = 1** $\Rightarrow$ **sin x =|sin y| = 1**

90. **Let** $f: R \to R$ **be differentiable at** $c \in R$ **and** $f(c) = 0$. **If** $g(x) = |f(x)|$, **then at** $x = c$, **g is**

1) not differentiable **2) differentiable if** $f'(c) = 0$

3) not differentiable if $f'(c) = 0$ **4) differentiable if** $f'(c) \neq 0$

Ans. **2**

Sol. $g'(x) = \frac{|f(x)|}{f(x)} f'(x)$ **If** $f'(c)$**=0 then** $g'(c^+) = g'(c^-) = 0$ $\therefore$ **g is differentiable if** $f'(c)$**=0**