1. Let \( f \) be a differentiable function from \( \mathbb{R} \) to \( \mathbb{R} \) such that \( |f(x) - f(y)| \leq 2|x - y|^{3/2} \), for all \( x, y \in \mathbb{R} \). If \( f(0) = 1 \), then \( \int_0^1 f^2(x)\,dx \) is equal to:

- (1) 0
- (2) \( \frac{1}{2} \)
- (3) 2
- (4) 1

**Ans. (4)**

**Sol.**

\[ |f(x) - f(y)| \leq 2|x - y|^{3/2} \]

Divide both sides by \( |x - y|^{1/2} \):

\[ \left| \frac{f(x) - f(y)}{x - y} \right| \leq 2|x - y| \]

Applying limit as \( x \to y \):

\[ |f'(y)| < 0 \]

\[ f'(y) = 0 \]

\[ f(x) = c \]

\[ f(x) = 1 \]

\[ \int_0^1 f(x)\,dx = 1 \]

2. If \( \int_0^{\pi/2} \frac{\tan \theta}{\sqrt{2}k \sec \theta} \, d\theta = 1 - \frac{1}{\sqrt{2}}, (k > 0) \), then the value of \( k \) is:

- (1) 2
- (2) \( \frac{1}{2} \)
- (3) 4
- (4) 1

**Ans. (1)**

**Sol.**

\[ \frac{1}{\sqrt{2}k} \int_0^{\pi/2} \frac{\tan \theta}{\sec \theta} \, d\theta = \frac{1}{\sqrt{2}k} \int_0^{\pi/2} \frac{\sin \theta}{\cos \theta} \, d\theta \]

\[ = \frac{1}{\sqrt{2}k} \left[ \frac{\sqrt{k}}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} \right) \right] \]

Given it is \( 1 - \frac{1}{\sqrt{2}} \Rightarrow k = 2 \)

3. The coefficient of \( t^4 \) in the expansion of

\[ \left( \frac{1-t^6}{1-t} \right)^3 \]

is:

- (1) 12
- (2) 15
- (3) 10
- (4) 14

**Ans. (2)**

**Sol.**

\[ (1-t^6)^3 (1-t)^{-3} \]

\[ (1-t^{18} - 3t^6 + 3t^{12}) (1-t)^{-3} \]

\[ \Rightarrow \text{coefficient of } t^4 \text{ in } (1-t)^{-3} \text{ is } 3^3 = 27 \]

4. For each \( x \in \mathbb{R} \), let \([x]\) be the greatest integer less than or equal to \( x \). Then

\[ \lim_{x \to 0^+} \frac{x([x]+[x])\sin[x]}{|x|} \]

is equal to:

- (1) \(-\sin 1\)
- (2) 0
- (3) 1
- (4) \(\sin 1\)

**Ans. (1)**

**Sol.**

\[ \lim_{x \to 0^+} \frac{x([x]+[x])\sin[x]}{|x|} \]

\[ x \to 0^+ \]

\[ [x] = 1 \Rightarrow \lim_{x \to 0^+} \frac{x(-x-1)\sin(-1)}{-x} = -\sin 1 \]

\[ |x| = -x \]

5. If both the roots of the quadratic equation \( x^2 - mx + 4 = 0 \) are real and distinct and they lie in the interval \([1,5]\), then \( m \) lies in the interval:

- (1) \((4,5)\)
- (2) \((3,4)\)
- (3) \((5,6)\)
- (4) \((-5,-4)\)

**Ans. (Bonus/1)**

**Sol.**

\( x^2 - mx + 4 = 0 \)

\( \alpha, \beta \in [1,5] \)

(1) \( D > 0 \Rightarrow m^2 - 16 > 0 \)

\[ \Rightarrow m \in (-\infty,-4) \cup (4,\infty) \]

(2) \( f(1) \geq 0 \Rightarrow 5-m \geq 0 \Rightarrow m \in (-\infty,5] \)

(3) \( f(5) \geq 0 \Rightarrow 29-5m \geq 0 \Rightarrow m \in \left(-\infty, \frac{29}{5}\right] \)

(4) \( 1 < \frac{-b}{2a} < 5 \Rightarrow 1 < \frac{m}{2} < 5 \Rightarrow m \in (2,10) \)

\[ \Rightarrow m \in (4,5) \]

No option correct : Bonus

* If we consider \( \alpha, \beta \in (1,5) \) then option (1) is correct.
6. If

\[
A = \begin{bmatrix}
e^{t} & e^{-t} \cos t & e^{-t} \sin t \\
e^{t} & -e^{-t} \cos t - e^{-t} \sin t & -e^{-t} \sin t + e^{-t} \cos t \\
e^{t} & 2e^{-t} \sin t & -2e^{-t} \cos t
\end{bmatrix}
\]

Then A is -

(1) Invertible only if \(t = \frac{\pi}{2}\)
(2) not invertible for any \(t \in \mathbb{R}\)
(3) invertible for all \(t \in \mathbb{R}\)
(4) invertible only if \(t = \pi\)

Ans. (3)

Sol. \[|A| = e^{-t} \begin{vmatrix} 1 & \cos t & \sin t \\ 2 \sin t & -2 \cos t & 1 \end{vmatrix} = e^{t}(5 \cos^2 t + 5 \sin^2 t) \quad \forall \ t \in \mathbb{R} = 5e^{t} \neq 0 \quad \forall \ t \in \mathbb{R}\]

7. The area of the region \(A = \{(x,y): 0 \leq y \leq x(x+1) \text{ and } -1 \leq x \leq 1\}\) in sq. units, is :

(1) \(\frac{2}{3}\)  (2) \(\frac{1}{3}\)  (3) 2  (4) \(\frac{4}{3}\)

Ans. (3)

Sol. The graph is as follows:

\[
\int_{-1}^{0} (-x^2 + 1) \, dx + \int_{0}^{1} (x^2 + 1) \, dx = 2
\]
11. The logical statement
\[ \neg (\neg p \lor q) \lor (p \land r) \land (\neg q \land r) \]
is equivalent to:
(1) \((p \land r) \land \neg q\)
(2) \(\neg p \land \neg q \land r\)
(3) \(\neg p \lor r\)
(4) \((p \land \neg q) \lor r\)

Ans. (1)

Sol. 
\[ s[\neg (\neg p \lor q) \land (p \land r) \land (\neg q \land r)] \cap (\neg q \land r) \]
\[ = [(p \land \neg q) \lor (p \land r)] \land (\neg q \land r) \]
\[ = [p \land (\neg q \lor r)] \land (\neg q \land r) \]
\[ = p \land (\neg q \land r) \]
\[ = (p \land r) \land \neg q \]

12. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red, is:

(1) \(\frac{26}{49}\)
(2) \(\frac{32}{49}\)
(3) \(\frac{27}{49}\)
(4) \(\frac{21}{49}\)

Ans. (2)

Sol. 
\[ E_1 : \text{Event of drawing a Red ball and placing a green ball in the bag} \]
\[ E_2 : \text{Event of drawing a green ball and placing a red ball in the bag} \]
\[ E : \text{Event of drawing a red ball in second draw} \]

\[ P(E) = P(E_1) \times P\left(\frac{E}{E_1}\right) + P(E_2) \times P\left(\frac{E}{E_2}\right) \]
\[ = \frac{5}{7} \times \frac{4}{7} + \frac{2}{7} \times \frac{6}{7} = \frac{32}{49} \]

13. If \(0 < x < \frac{\pi}{2}\), then the number of values of \(x\) for which \(\sin x - \sin2x + \sin3x = 0\), is:
(1) 2
(2) 1
(3) 3
(4) 4

Ans. (1)

Sol. 
\[ \sin x - \sin2x + \sin3x = 0 \]
\[ \Rightarrow (\sin x + \sin3x) - \sin2x = 0 \]
\[ \Rightarrow 2\sin x \cos x - \sin2x = 0 \]
\[ \Rightarrow \sin2x(2\cos x - 1) = 0 \]
\[ \Rightarrow \sin2x = 0 \text{ or } \cos x = \frac{1}{2} \]
\[ \Rightarrow x = 0, \frac{\pi}{3} \]

14. The equation of the plane containing the straight line \(\frac{x}{2} = \frac{y}{3} = \frac{z}{4}\) and perpendicular to the plane containing the straight lines
\[ \frac{x}{3} = \frac{y}{4} = \frac{z}{2} \quad \text{and} \quad \frac{x}{4} = \frac{y}{2} = \frac{z}{3} \]
is:
(1) \(x + 2y - 2z = 0\)
(2) \(x - 2y + z = 0\)
(3) \(5x + 2y - 4z = 0\)
(4) \(3x + 2y - 3z = 0\)

Ans. (2)
Sol. Vector along the normal to the plane containing the lines
\[
\frac{x}{3} = \frac{y}{4} = \frac{z}{2} \quad \text{and} \quad \frac{x}{4} = \frac{y}{2} = \frac{z}{3}
\]
is \(8\hat{i} - \hat{j} - 10\hat{k}\)

vector perpendicular to the vectors \(2\hat{i} + 3\hat{j} + 4\hat{k}\) and \(8\hat{i} - \hat{j} - 10\hat{k}\) is \(26\hat{i} - 52\hat{j} + 26\hat{k}\)

so, required plane is
\[26x - 52y + 26z = 0\]
\[x - 2y + z = 0\]

15. Let the equations of two sides of a triangle be \(3x - 2y + 6 = 0\) and \(4x + 5y - 20 = 0\). If the orthocentre of this triangle is at (1,1), then the equation of its third side is :

(1) \(122y - 26x - 1675 = 0\)
(2) \(26x + 61y + 1675 = 0\)
(3) \(122y + 26x + 1675 = 0\)
(4) \(26x - 122y - 1675 = 0\)

Ans. (4)

Sol. Equation of AB is
\[3x - 2y + 6 = 0\]
equation of AC is
\[4x + 5y - 20 = 0\]
Equation of BE is
\[2x + 3y - 5 = 0\]
Equation of CF is \(5x - 4y - 1 = 0\)

\[⇒ \text{Equation of BC is } 26x - 122y = 1675\]

16. If \(x = 3 \tan t\) and \(y = 3 \sec t\), then the value of \(\frac{d^2y}{dx^2}\) at \(t = \frac{\pi}{4}\), is:

(1) \(\frac{3}{2\sqrt{2}}\)
(2) \(\frac{1}{3\sqrt{2}}\)
(3) \(\frac{1}{6}\)
(4) \(\frac{1}{6\sqrt{2}}\)

Ans. (4)

Sol.
\[
\frac{dx}{dt} = 3\sec^2 t \\
\frac{dy}{dt} = 3\sec t \tan t \\
\frac{dy}{dx} = \tan t \\
\frac{dx}{\sec t} = \cos t \\
\frac{d^2y}{dx^2} = \frac{\cos^3 t}{3\sec^2 t} = \frac{\cos^3 t}{3} = \frac{1}{3.2\sqrt{2}} = \frac{1}{6\sqrt{2}}
\]
20. A hyperbola has its centre at the origin, passes through the point (4,2) and has transverse axis of length 4 along the x-axis. Then the eccentricity of the hyperbola is:

\( \frac{2}{\sqrt{3}} \) (1) \( \frac{3}{2} \) (2) \( \sqrt{3} \) (3) 2 (4) 4

Ans. (1)

Sol. \[ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \]
\[ 2a = 4 \quad a = 2 \]
\[ \frac{x^2}{4} - \frac{y^2}{b^2} = 1 \]

Passes through (4,2)
\[ 4 - \frac{4}{b^2} = 1 \Rightarrow b^2 = \frac{3}{2} \Rightarrow e = \frac{2}{\sqrt{3}} \]

21. Let \( A = \{ x \in \mathbb{R} : x \text{ is not a positive integer} \} \)

Define a function \( f : A \rightarrow \mathbb{R} \) as \( f(x) = \frac{2x}{x - 1} \) then \( f \) is
(1) injective but not surjective
(2) not injective
(3) surjective but not injective
(4) neither injective nor surjective

Ans. (1)

Sol. \[ f(x) = 2\left(1 + \frac{1}{x - 1}\right) \]
\[ f'(x) = -\frac{2}{(x - 1)^2} \]
\[ \Rightarrow f \text{ is one-one but not onto} \]

22. If \[ f(x) = \int \frac{5x^8 + 7x^6}{x^2 + 2x^4 + 3x^2 + 4x + 5} \text{dx}, \] and \( f(0) = 0 \), then the value of \( f(1) \) is:

\( -\frac{1}{2} \) (1) \( \frac{1}{2} \) (2) \( -\frac{1}{4} \) (3) \( \frac{1}{4} \) (4) \( \frac{1}{4} \)

Ans. (4)

Sol. \[ \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^4)^2} \text{dx} = \int \frac{1}{1 + \frac{1}{x^2} + \frac{1}{x^4}} \text{dx} = \frac{1}{2 + \frac{1}{x^2} + \frac{1}{x^4}} + C \]

As \( f(0) = 0 \), \( f(x) = \frac{x^7}{2x^2 + x^2 + 1} \)
\[ f(1) = \frac{1}{4} \]

23. If the circles \( x^2 + y^2 + 16x - 20y + 164 = r^2 \) and \( (x-4)^2 + (y-7)^2 = 36 \) intersect at two distinct points, then:
(1) \( 0 < r < 1 \) (2) \( 1 < r < 11 \) (3) \( r > 11 \) (4) \( r = 11 \)

Ans. (2)

Sol. \( x^2 + y^2 - 16x - 20y + 164 = r^2 \)
A(8,10), \( R_1 = r \)
\( (x - 4)^2 + (y - 7)^2 = 36 \)
B(4,7), \( R_2 = 6 \)
\[ |R_1 - R_2| < AB < R_1 + R_2 \]
\[ \Rightarrow 1 < r < 11 \]

24. Let \( S \) be the set of all triangles in the \( xy \)-plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in \( S \) has area 50 sq. units, then the number of elements in the set \( S \) is:

(1) 9 (2) 18 (3) 32 (4) 36

Ans. (4)

Sol. Let \( A(\alpha,0) \) and \( B(0,\beta) \)
be the vectors of the given triangle AOB
\[ \Rightarrow |\alpha\beta| = 100 \]
\[ \Rightarrow \text{Number of triangles} \]
\[ = 4 \times \text{(number of divisors of 100)} \]
\[ = 4 \times 9 = 36 \]

25. The sum of the following series
\[ 1 + 6 + \frac{9(1^2 + 2^2 + 3^2)}{7} + \frac{12(1^2 + 2^2 + 3^2 + 4^2)}{9} + \frac{15(1^2 + 2^2 + \ldots + 5^2)}{11} + \ldots \text{up to 15 terms}, \text{is:} \]

(1) 7820 (2) 7830 (3) 7520 (4) 7510

Ans. (1)
### 27. Problem Statement

If the system of linear equations

\[
\begin{align*}
3y - 5z &= h \\
-2x + 5y - 9z &= k
\end{align*}
\]

is consistent, then:

1. \(g + h + k = 0\)
2. \(2g + h + k = 0\)
3. \(g + h + 2k = 0\)
4. \(g + 2h + k = 0\)

**Solution**

The system is consistent, so the given options do not hold true.

**Answer**

(2)

### 28. Problem Statement

Let \(f(x) = f(x) \cdot f(y)\) for all \(x, y, \in [0,1]\), and \(f(0) \neq 0\). If \(y = y(x)\) satisfies the differential equation, \(\frac{dy}{dx} = f(x)\) with \(y(0) = 1\), then

\[
\frac{dy}{dx} = f(x)
\]

is equal to:

1. \(\frac{1}{2}\)
2. \(4\)
3. \(2\)
4. \(\frac{7}{13}\)

**Solution**

Given \(f(xy) = f(x) \cdot f(y)\) and \(f(0) = 1\), we can integrate to find \(y\).

\[
\frac{dy}{dx} = f(x)
\]

At \(x = 0, y = 1\), then \(y = x + c\), where \(c = 1\), and \(y = x + 1\), which is consistent with the differential equation.

**Answer**

(2)

### 29. Problem Statement

A data consists of \(n\) observations:

\[
x_1, x_2, \ldots, x_n.
\]

If \(\sum_{i=1}^{n}(x_i + 1)^2 = 9n\) and \(\sum_{i=1}^{n}(x_i - 1)^2 = 5n\), then the standard deviation of this data is:

1. \(5\)
2. \(\sqrt{5}\)
3. \(\sqrt{7}\)
4. \(2\)

**Solution**

Using the given formulas, we can calculate the standard deviation.

**Answer**

(2)
Sol. \[ \sum (x_i + 1)^2 = 9n \] \[ \sum (x_i - 1)^2 = 5n \] 

(1) + (2) \[ \Rightarrow \sum (x_i^2 + 1) = 7n \] 

\[ \Rightarrow \frac{\sum x_i^2}{n} = 6 \] 

(1) - (2) \[ \Rightarrow 4\Sigma x_i = 4n \] 

\[ \Rightarrow \Sigma x_i = n \] 

\[ \Rightarrow \frac{\Sigma x_i}{n} = 1 \] 

\[ \Rightarrow \text{variance} = 6 - 1 = 5 \] 

\[ \Rightarrow \text{Standard deviation} = \sqrt{5} \]

30. The number of natural numbers less than 7,000 which can be formed by using the digits 0, 1, 3, 7, 9 (repetition of digits allowed) is equal to:

(1) 250 (2) 374 (3) 372 (4) 375

Ans. (2)

Sol. \[
\begin{array}{c|c|c}
\text{a}_1 & \text{a}_2 & \text{a}_3 \\
\end{array}
\]

Number of numbers = \(5^3 - 1\)

\[
\begin{array}{c|c|c|c}
\text{a}_3 & \text{a}_1 & \text{a}_2 & \text{a}_3 \\
\end{array}
\]

2 ways for \(a_3\)

Number of numbers = \(2 \times 5^3\)

Required number = \(5^3 + 2 \times 5^3 - 1\)

\[= 374\]