TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On SATURDAY 12th JANUARY., 2019) TIME : 02 : 30 PM To 05 : 30 PM

MATHEMATICS

1. Let Z be the set of integers. If $A=\{x\in Z:2(x+2)(x^2-5x+6)\}=1$ and $B=\{x\in Z:-3<2x-1<9\}$, then the number of subsets of the set A × B, is :

(1) 2^{18} (2) 2^{10} (3) 2^{15} (4) 2^{12}

Ans. (3)

2. If $\sin^4\alpha+4\cos^4\beta+2=4\sqrt{2}\sin\alpha\cos\beta$; $\alpha,\beta\in[0,\pi]$, then $\cos(\alpha+\beta)-\cos(\alpha-\beta)$ is equal to :

(1) 0 (2) $-\sqrt{2}$ (3) –1 (4) $\sqrt{2}$

Ans. (2)

3. If an angle between the line, $\frac{x+1}{2}=\frac{y-2}{1}=\frac{z-3}{-2}$ and the plane, $x-2y-kz=3$ is $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$, then a value of k is:

(1) $-\frac{5}{3}$ (2) $\sqrt{\frac{3}{5}}$ (3) $\sqrt{\frac{5}{3}}$ (4) $-\frac{3}{5}$

Ans. (3)

4. If a straight line passing thourgh the point P(–3, 4) is such that its intercepted portion between the coordinate axes is bisected at P, then its equation is :

(1) $x-y+7=0$

(2) $3x-4y+25=0$

(3) $4x+3y=0$

(4) $4x-3y+24=0$

Ans. (4)

5. The integral $\int\frac{3x^{13}+2x^{11}}{(2x^4+3x^2+1)^4}dx$ is equal to :

(where C is a constant of integration)

(1) $\frac{x^4}{(2x^4+3x^2+1)^3}+C$

(2) $\frac{x^{12}}{6(2x^4+3x^2+1)^3}+C$

(3) $\frac{x^4}{6(2x^4+3x^2+1)^3}+C$

(4) $\frac{x^{12}}{(2x^4+3x^2+1)^3}+C$

Ans. (2)

6. There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is :

(1) 9 (2) 11 (3) 12 (4) 7

Ans. (3)

7. If the function *f* given by $f(x)=x^3-3(a-2)x^2+3ax+7$, for some $a\in R$ is increasing in (0, 1] and decreasing in [1, 5), then a root of the equation, $\frac{f(x)-14}{(x-1)^2}=0(x\neq1)$ is :

(1) 6 (2) 5 (3) 7 (4) –7

Ans. (3)

8. Let *f* be a differentiable function such that $f(1)=2$ and $f'(x)=f(x)$ for all $x\in R$. If $h(x)=f(f(x))$, then $h'(1)$ is equal to :

(1) 4e (2) $4e^2$ (3) 2e (4) $2e^2$

Ans. (1)

9. The tangent to the curve $y = x^2 - 5x + 5$, parallel to the line $2y = 4x + 1$, also passes through the point.

(1) $\left(\frac{1}{4},\frac{7}{2}\right)$ (2) $\left(\frac{7}{2},\frac{1}{4}\right)$

(3) $\left(-\frac{1}{8},7\right)$ (4) $\left(\frac{1}{8},-7\right)$

Ans. (4)

10. Let S be the set of all real values of λ such that a plane passing through the points $(-\lambda^2, 1, 1)$, $(1, -\lambda^2, 1)$ and $(1, 1, -\lambda^2)$ also passes through the point $(-1, -1, 1)$. Then S is equal to :

(1) $\{\sqrt{3}\}$ (2) $\{\sqrt{3}-\sqrt{3}\}$

(3) $\{1, -1\}$ (4) $\{3, -3\}$

Ans. (2)

11. If a circle of radius R passes through the origin O and intersects the coordinate axes at A and B, then the locus of the foot of perpendicular from O on AB is :

(1) $(x^2 + y^2)^2 = 4Rx^2y^2$

(2) $(x^2 + y^2)(x + y) = R^2xy$

(3) $(x^2 + y^2)^3 = 4R^2x^2y^2$

(4) $(x^2 + y^2)^2 = 4R^2x^2y^2$

Ans. (3)

12. The equation of a tangent to the parabola, $x^2 = 8y$, which makes an angle θ with the positive direction of x-axis, is :

(1) $x = y\cot\theta + 2\tan\theta$ (2) $x = y\cot\theta - 2\tan\theta$

(3) $y = x\tan\theta - 2\cot\theta$ (4) $y = x\tan\theta + 2\cot\theta$

Ans. (1)

13. If the angle of elevation of a cloud from a point P which is 25 m above a lake be 30º and the angle of depression of reflection of the cloud in the lake from P be 60º, then the height of the cloud (in meters) from the surface of the lake is :

(1) 42 (2) 50 (3) 45 (4) 60

Ans. (2)

14. The integral $\int_1^e \left\{\left(\frac{x}{e}\right)^{2x} - \left(\frac{e}{x}\right)^x\right\}\log_e x\,dx$ is equal to :

(1) $\frac{1}{2}-e-\frac{1}{e^2}$ (2) $\frac{3}{2}-\frac{1}{e}-\frac{1}{2e^2}$

(3) $-\frac{1}{2}+\frac{1}{e}-\frac{1}{2e^2}$ (4) $\frac{3}{2}-e-\frac{1}{2e^2}$

Ans. (4)

15. $\lim\limits_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+....+\frac{1}{5n}\right)$ is equal to :

(1) $\frac{\pi}{4}$ (2) $\tan^{-1}(2)$

(3) $\tan^{-1}(3)$ (4) $\frac{\pi}{2}$

Ans. (2)

16. The set of all values of λ for which the system of linear equations.

$x - 2y - 2z = \lambda x$

$x + 2y + z = \lambda y$

$-x - y = \lambda z$

has a non-trivial solution.

(1) contains more than two elements

(2) is a singleton

(3) is an empty set

(4) contains exactly two elements

Ans. (2)

17. If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P., then n can be:

(1) 14 (2) 11 (3) 9 (4) 12

Ans. (1)

18. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three unit vectors, out of which vectors $\vec{b}$ and $\vec{c}$ are non-parallel. If α and β are the angles which vector $\vec{a}$ makes with vectors $\vec{b}$ and $\vec{c}$ respectively and $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\frac{1}{2}\vec{b}$, then $|\alpha-\beta|$ is equal to :

(1) 60º (2) 30º (3) 90º (4) 45º

Ans. (2)

19. If $A=\begin{bmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{bmatrix}$; then for all $\theta\in\left(\frac{3\pi}{4},\frac{5\pi}{4}\right)$, det(A) lies in the interval :

(1) $\left[\frac{5}{2},4\right)$ (2) $\left(\frac{3}{2},3\right]$

(3) $\left(0,\frac{3}{2}\right]$ (4) $\left(1,\frac{5}{2}\right]$

Ans. (2)

20. $\lim\limits_{x\to 1-}\dfrac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$ ie equal to :

(1) $\dfrac{1}{\sqrt{2\pi}}$ (2) $\sqrt{\dfrac{\pi}{2}}$ (3) $\sqrt{\dfrac{2}{\pi}}$ (4) $\sqrt{\pi}$

Ans. (3)

21. The expression ~(~p→q) is logically equvalent to :

(1) ~ p ^ ~ q (2) p ^ q

(3) ~ p ^ q (4) p ^ ~ q

Ans. (1)

22. The total number of irrational terms in the binomial expansion of $\left(7^{1/5}-3^{1/10}\right)^{60}$ is :

(1) 55 (2) 49 (3) 48 (4) 54

Ans. (4)

23. The mean and the variance of five observation are 4 and 5.20, respectively. If three of the observations are 3, 4 and 4; then then absolute value of the difference of the other two observations, is :

(1) 1 (2) 3 (3) 7 (4) 5

Ans. (3)

24. If the sum of the first 15 tems of the series $\left(\dfrac{3}{4}\right)^3+\left(1\dfrac{1}{2}\right)^3+\left(2\dfrac{1}{4}\right)^3+3^3+\left(3\dfrac{3}{4}\right)^3+....$ is equal to 225 k, then k is equal to :

(1) 9 (2) 27 (3) 108 (4) 54

Ans. (2)

25. Let S and S' be the foci of the ellipse and B be any one of the extremities of its minor axis. If ΔS'BS is a right angled triangle with right angle at B and area (ΔS'BS) = 8 sq. units, then the length of a latus rectum of the ellipse is :

(1) $2\sqrt{2}$ (2) 2 (3) 4 (4) $4\sqrt{2}$

Ans. (3)

26. In a class of 60 students, 40 opted for NCC, 30 opted for NSS and 20 opted for both NCC and NSS. If one of these students is selected at random, then the probability that the student selected has opted neither for NCC nor for NSS is :

(1) $\dfrac{2}{3}$ (2) $\dfrac{1}{6}$ (3) $\dfrac{1}{3}$ (4) $\dfrac{5}{6}$

Ans. (2)

27. The number of integral values of m for which the quadratic expression.

$(1 + 2m)x^2 - 2(1 + 3m)x + 4(1 + m)$, $x\in R$, is always positive, is :

(1) 8 (2) 7 (3) 6 (4) 3

Ans. (2)

28. In a game, a man wins Rs. 100 if he gets 5 of 6 on a throw of a fair die and loses Rs. 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws, then his expected gain/loss (in rupees) is :

(1) $\dfrac{400}{3}$ gain (2) $\dfrac{400}{3}$ loss

(3) 0 (4) $\dfrac{400}{9}$ loss

Ans. (3)

29. If a cuver passes through the point (1, –2) and has slope of the tangent at any point (x, y) on it as $\dfrac{x^2-2y}{x}$, then the curve also passes through the point :

(1) $\left(-\sqrt{2},1\right)$ (2) $\left(\sqrt{3},0\right)$

(3) (–1, 2) (4) (3, 0)

Ans. (2)

30. Let Z_1 and Z_2 be two complex numbers satisfying $|Z_1| = 9$ and $|Z_2-3-4i|=4$. Then the minimum value of $|Z_1-Z_2|$ is :

(1) 0 (2) 1 (3) $\sqrt{2}$ (4) 2

Ans. (1)