

TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Friday 11th JANUARY, 2019) TIME : 9 : 30 AM To 12 : 30 PM

MATHEMATICS

1. Let $A = \begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. If $AA^T = I_3$, then $|p|$

is :

- (1) $\frac{1}{\sqrt{2}}$
- (2) $\frac{1}{\sqrt{5}}$
- (3) $\frac{1}{\sqrt{6}}$
- (4) $\frac{1}{\sqrt{3}}$

Ans. (1)

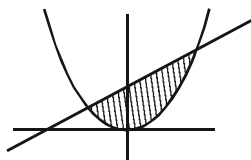
Sol. A is orthogonal matrix

$$\Rightarrow 0^2 + p^2 + p^2 = 1 \Rightarrow |p| = \frac{1}{\sqrt{2}}$$

2. The area (in sq. units) of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$:-

- (1) $\frac{5}{4}$
- (2) $\frac{9}{8}$
- (3) $\frac{3}{4}$
- (4) $\frac{7}{8}$

Ans. (2)



Sol.

$$\begin{aligned} x &= 4y - 2 \text{ \& } x^2 = 4y \\ \Rightarrow x^2 &= x + 2 \Rightarrow x^2 - x - 2 = 0 \\ x &= 2, -1 \end{aligned}$$

$$\text{So, } \int_{-1}^2 \left(\frac{x+2}{4} - \frac{x^2}{4} \right) dx = \frac{9}{8}$$

3. The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2} - d$ each, 10 items gave outcome $\frac{1}{2}$ each and the remaining

10 items gave outcome $\frac{1}{2} + d$ each. If the variance of this outcome data is $\frac{4}{3}$ then $|d|$ equals :-

- (1) 2
- (2) $\frac{\sqrt{5}}{2}$
- (3) $\frac{2}{3}$
- (4) $\sqrt{2}$

Ans. (4)

Sol. Variance is independent of origin. So we shift the given data by $\frac{1}{2}$.

$$\text{so, } \frac{10d^2 + 10 \times 0^2 + 10d^2}{30} - (0)^2 = \frac{4}{3}$$

$$\Rightarrow d^2 = 2 \Rightarrow |d| = \sqrt{2}$$

4. The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is :

- (1) $\frac{4}{9}$
- (2) $\frac{2}{9}$
- (3) $\frac{2}{3}$
- (4) $\frac{1}{3}$

Ans. (3)

$$\text{Sol. } \frac{a}{1-r} = 3 \quad \dots(1)$$

$$\frac{a^3}{1-r^3} = \frac{27}{19} \Rightarrow \frac{27(1-r)^3}{1-r^3} = \frac{27}{19}$$

$$\Rightarrow 6r^2 - 13r + 6 = 0$$

$$\Rightarrow r = \frac{2}{3} \text{ as } |r| < 1$$

5. Let $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + \lambda\hat{j} + 4\hat{k}$ and $\vec{c} = 2\hat{i} + 4\hat{j} + (\lambda^2 - 1)\hat{k}$ be coplanar vectors.

Then the non-zero vector $\vec{a} \times \vec{c}$ is :

(1) $-14\hat{i} - 5\hat{j}$ (2) $-10\hat{i} - 5\hat{j}$

(3) $-10\hat{i} + 5\hat{j}$ (4) $-14\hat{i} + 5\hat{j}$

Ans. (3)

Sol. $[\vec{a} \ \vec{b} \ \vec{c}] = 0$

$$\Rightarrow \begin{vmatrix} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 2 & 4 & \lambda^2 - 1 \end{vmatrix} = 0$$

$$\Rightarrow \lambda^3 - 2\lambda^2 - 9\lambda + 18 = 0$$

$$\Rightarrow \lambda^2(\lambda - 2) - 9(\lambda - 2) = 0$$

$$\Rightarrow (\lambda - 3)(\lambda + 3)(\lambda - 2) = 0$$

$$\Rightarrow \lambda = 2, 3, -3$$

So, $\lambda = 2$ (as \vec{a} is parallel to \vec{c} for $\lambda = \pm 3$)

$$\text{Hence } \vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{vmatrix}$$

$$= -10\hat{i} + 5\hat{j}$$

6. Let $\left(-2 - \frac{1}{3}i\right)^3 = \frac{x + iy}{27}$ ($i = \sqrt{-1}$), where x

and y are real numbers, then $y - x$ equals :

(1) -85 (2) 85

(3) -91 (4) 91

Ans. (4)

$$\text{Sol. } \left(-2 - \frac{i}{3}\right)^3 = -\frac{(6+i)^3}{27}$$

$$= \frac{-198 - 107i}{27} = \frac{x + iy}{27}$$

Hence, $y - x = 198 - 107 = 91$

7. Let $f(x) = \begin{cases} -1, & -2 \leq x < 0 \\ x^2 - 1, & 0 \leq x \leq 2 \end{cases}$ and

$g(x) = |f(x)| + f(|x|)$. Then, in the interval

$(-2, 2)$, g is :-

(1) differentiable at all points

(2) not differentiable at two points

(3) not continuous

(4) not differentiable at one point

Ans. (4)

$$\text{Sol. } |f(x)| = \begin{cases} 1 & , \quad -2 \leq x < 0 \\ 1 - x^2 & , \quad 0 \leq x < 1 \\ x^2 - 1 & , \quad 1 \leq x \leq 2 \end{cases}$$

and $f(|x|) = x^2 - 1, x \in [-2, 2]$

$$\text{Hence } g(x) = \begin{cases} x^2 & , \quad x \in [-2, 0) \\ 0 & , \quad x \in [0, 1) \\ 2(x^2 - 1) & , \quad x \in [1, 2] \end{cases}$$

It is not differentiable at $x = 1$

8. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \frac{x}{1 + x^2}$,

$x \in \mathbb{R}$. Then the range of f is :

(1) $(-1, 1) - \{0\}$ (2) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(3) $\mathbb{R} - \left[-\frac{1}{2}, \frac{1}{2}\right]$ (4) $\mathbb{R} - [-1, 1]$

Ans. (2)

Sol. $f(0) = 0$ & $f(x)$ is odd.

Further, if $x > 0$ then

$$f(x) = \frac{1}{x + \frac{1}{x}} \in \left(0, \frac{1}{2}\right]$$

$$\text{Hence, } f(x) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

9. The sum of the real values of x for which the middle term in the binomial expansion of

$$\left(\frac{x^3}{3} + \frac{3}{x}\right)^8 \text{ equals } 5670 \text{ is :}$$

- (1) 6 (2) 8 (3) 0 (4) 4

Ans. (3)

Sol. $T_5 = {}^8C_4 \frac{x^{12}}{81} \times \frac{81}{x^4} = 5670$

$$\Rightarrow 70x^8 = 5670$$

$$\Rightarrow x = \pm\sqrt{3}$$

10. The value of r for which

$${}^{20}C_r {}^{20}C_0 + {}^{20}C_{r-1} {}^{20}C_1 + {}^{20}C_{r-2} {}^{20}C_2 + \dots + {}^{20}C_0 {}^{20}C_r$$

is maximum, is

- (1) 20 (2) 15
(3) 11 (4) 10

Ans. (1)

Sol. Given sum = coefficient of x^r in the expansion of $(1+x)^{20}(1+x)^{20}$, which is equal to ${}^{40}C_r$

It is maximum when $r = 20$

11. Let a_1, a_2, \dots, a_{10} be a G.P. If $\frac{a_3}{a_1} = 25$, then

$$\frac{a_9}{a_5} \text{ equals :}$$

- (1) $2(5^2)$ (2) $4(5^2)$
(3) 5^4 (4) 5^3

Ans. (3)

Sol. a_1, a_2, \dots, a_{10} are in G.P.,

Let the common ratio be r

$$\frac{a_3}{a_1} = 25 \Rightarrow \frac{a_1 r^2}{a_1} = 25 \Rightarrow r^2 = 25$$

$$\frac{a_9}{a_5} = \frac{a_1 r^8}{a_1 r^4} = r^4 = 5^4$$

12. If $\int \frac{\sqrt{1-x^2}}{x^4} dx = A(x)(\sqrt{1-x^2})^m + C$, for

a suitable chosen integer m and a function $A(x)$, where C is a constant of integration then $(A(x))^m$ equals :

(1) $\frac{-1}{3x^3}$ (2) $\frac{-1}{27x^9}$

(3) $\frac{1}{9x^4}$ (4) $\frac{1}{27x^6}$

Ans. (2)

Sol. $\int \frac{\sqrt{1-x^2}}{x^4} dx = A(x)(\sqrt{1-x^2})^m + C$

$$\int \frac{|x| \sqrt{\frac{1}{x^2} - 1}}{x^4} dx,$$

Put $\frac{1}{x^2} - 1 = t \Rightarrow \frac{dt}{dx} = \frac{-2}{x^3}$

Case-I $x \geq 0$

$$-\frac{1}{2} \int \sqrt{t} dt \Rightarrow -\frac{t^{3/2}}{3} + C$$

$$\Rightarrow -\frac{1}{3} \left(\frac{1}{x^2} - 1\right)^{3/2}$$

$$\Rightarrow \frac{(\sqrt{1-x^2})^3}{-3x^2} + C$$

$$A(x) = -\frac{1}{3x^3} \text{ and } m = 3$$

$$(A(x))^m = \left(-\frac{1}{3x^3}\right)^3 = -\frac{1}{27x^9}$$

Case-II $x \leq 0$

We get $\frac{(\sqrt{1-x^2})^3}{-3x^3} + C$

$$A(x) = \frac{1}{-3x^3}, \quad m = 3$$

$$(A(x))^m = \frac{-1}{27x^9}$$

18. Let $[x]$ denote the greatest integer less than or equal to x . Then :-

$$\lim_{x \rightarrow 0} \frac{\tan(\pi \sin^2 x) + (|x| - \sin(x[x]))^2}{x^2}$$

- (1) equals π
 (2) equals 0
 (3) equals $\pi + 1$
 (4) does not exist

Ans. (4)

Sol. R.H.L. = $\lim_{x \rightarrow 0^+} \frac{\tan(\pi \sin^2 x) + (|x| - \sin(x[x]))^2}{x^2}$

(as $x \rightarrow 0^+ \Rightarrow [x] = 0$)

$$= \lim_{x \rightarrow 0^+} \frac{\tan(\pi \sin^2 x) + x^2}{x^2}$$

$$= \lim_{x \rightarrow 0^+} \frac{\tan(\pi \sin^2 x)}{(\pi \sin^2 x)} + 1 = \pi + 1$$

L.H.L. = $\lim_{x \rightarrow 0^-} \frac{\tan(\pi \sin^2 x) + (-x + \sin x)^2}{x^2}$

(as $x \rightarrow 0^- \Rightarrow [x] = -1$)

$$\lim_{x \rightarrow 0^+} \frac{\tan(\pi \sin^2 x)}{\pi \sin^2 x} \cdot \frac{\pi \sin^2 x}{x^2} + \left(-1 + \frac{\sin x}{x}\right)^2 \Rightarrow \pi$$

R.H.L. \neq L.H.L.

19. The direction ratios of normal to the plane through the points $(0, -1, 0)$ and $(0, 0, 1)$ and

making an angle $\frac{\pi}{4}$ with the plane $y - z + 5 = 0$ are:

- (1) $2\sqrt{3}, 1, -1$
 (2) $2, \sqrt{2}, -\sqrt{2}$
 (3) $2, -1, 1$
 (4) $\sqrt{2}, 1, -1$

Ans. (2, 4)

Sol. Let the equation of plane be

$$a(x - 0) + b(y + 1) + c(z - 0) = 0$$

It passes through $(0, 0, 1)$ then

$$b + c = 0 \quad \dots(1)$$

$$\text{Now } \cos \frac{\pi}{4} = \frac{a(0) + b(1) + c(-1)}{\sqrt{2}\sqrt{a^2 + b^2 + c^2}}$$

$$\Rightarrow a^2 = -2bc \text{ and } b = -c$$

$$\text{we get } a^2 = 2c^2$$

$$\Rightarrow a = \pm\sqrt{2}c$$

$$\Rightarrow \text{direction ratio } (a, b, c) = (\sqrt{2}, -1, 1) \text{ or}$$

$$(\sqrt{2}, 1, -1)$$

20. If $x \log_e(\log_e x) - x^2 + y^2 = 4$ ($y > 0$), then dy/dx at $x = e$ is equal to :

(1) $\frac{e}{\sqrt{4 + e^2}}$

(2) $\frac{(1 + 2e)}{2\sqrt{4 + e^2}}$

(3) $\frac{(2e - 1)}{2\sqrt{4 + e^2}}$

(4) $\frac{(1 + 2e)}{\sqrt{4 + e^2}}$

Ans. (3)

Sol. Differentiating with respect to x ,

$$x \cdot \frac{1}{\ln x} \cdot \frac{1}{x} + \ln(\ln x) - 2x + 2y \cdot \frac{dy}{dx} = 0$$

at $x = e$ we get

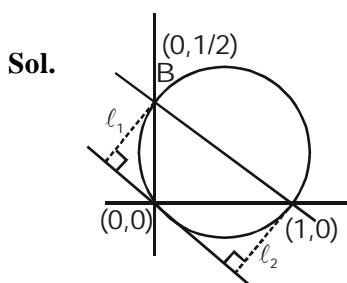
$$1 - 2e + 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{2e - 1}{2y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2e - 1}{2\sqrt{4 + e^2}} \text{ as } y(e) = \sqrt{4 + e^2}$$

21. The straight line $x + 2y = 1$ meets the coordinate axes at A and B. A circle is drawn through A, B and the origin. Then the sum of perpendicular distances from A and B on the tangent to the circle at the origin is :

- (1) $\frac{\sqrt{5}}{4}$
- (2) $\frac{\sqrt{5}}{2}$
- (3) $2\sqrt{5}$
- (4) $4\sqrt{5}$

Ans. (2)



Equation of circle

$$(x - 1)(x - 0) + (y - 0)\left(y - \frac{1}{2}\right) = 0$$

$$\Rightarrow x^2 + y^2 - x - \frac{y}{2} = 0$$

Equation of tangent of origin is $2x + y = 0$

$$l_1 + l_2 = \frac{2}{\sqrt{5}} + \frac{1}{2\sqrt{5}}$$

$$= \frac{4+1}{2\sqrt{5}} = \frac{\sqrt{5}}{2}$$

22. If q is false and $p \wedge q \leftrightarrow r$ is true, then which one of the following statements is a tautology?

- (1) $(p \vee r) \rightarrow (p \wedge r)$
- (2) $p \vee r$
- (3) $p \wedge r$
- (4) $(p \wedge r) \rightarrow (p \vee r)$

Ans. (4)

Sol. Given q is F and $(p \wedge q) \leftrightarrow r$ is T
 $\Rightarrow p \wedge q$ is F which implies that r is F
 $\Rightarrow q$ is F and r is F
 $\Rightarrow (p \wedge r)$ is always F
 $\Rightarrow (p \wedge r) \rightarrow (p \vee r)$ is tautology.

23. If $y(x)$ is the solution of the differential equation

$$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}, \quad x > 0,$$

where $y(1) = \frac{1}{2}e^{-2}$, then :

- (1) $y(x)$ is decreasing in $(0, 1)$
- (2) $y(x)$ is decreasing in $\left(\frac{1}{2}, 1\right)$
- (3) $y(\log_e 2) = \frac{\log_e 2}{4}$
- (4) $y(\log_e 2) = \log_e 4$

Ans. (2)

Sol.
$$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}$$

$$\text{I.F.} = e^{\int \left(\frac{2x+1}{x}\right) dx} = e^{\int \left(2 + \frac{1}{x}\right) dx} = e^{2x + \ln x} = e^{2x} \cdot x$$

$$\text{So, } y(xe^{2x}) = \int e^{-2x} \cdot xe^{2x} + C$$

$$\Rightarrow xye^{2x} = \int x dx + C$$

$$\Rightarrow 2xye^{2x} = x^2 + 2C$$

It passes through $\left(1, \frac{1}{2}e^{-2}\right)$ we get $C = 0$

$$y = \frac{xe^{-2x}}{2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}e^{-2x}(-2x+1)$$

$\Rightarrow f(x)$ is decreasing in $\left(\frac{1}{2}, 1\right)$

$$y(\log_e 2) = \frac{(\log_e 2)e^{-2(\log_e 2)}}{2}$$

$$= \frac{1}{8} \log_e 2$$

24. The maximum value of the function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set

$$S = \{x \in \mathbb{R} : x^2 + 30 \leq 11x\} \text{ is :}$$

- (1) 122 (2) -222
(3) -122 (4) 222

Ans. (1)

Sol. $S = \{x \in \mathbb{R}, x^2 + 30 - 11x \leq 0\}$
 $= \{x \in \mathbb{R}, 5 \leq x \leq 6\}$

Now $f(x) = 3x^3 - 18x^2 + 27x - 40$

$$\Rightarrow f'(x) = 9(x-1)(x-3),$$

which is positive in $[5, 6]$

$$\Rightarrow f(x) \text{ increasing in } [5, 6]$$

Hence maximum value $= f(6) = 122$

25. If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is

- (1) -81 (2) 100 (3) -300 (4) 144

Ans. (3)

Sol. $81x^2 + kx + 256 = 0$; $x = \alpha, \alpha^3$

$$\Rightarrow \alpha^4 = \frac{256}{81} \Rightarrow \alpha = \pm \frac{4}{3}$$

$$\text{Now } -\frac{k}{81} = \alpha + \alpha^3 = \pm \frac{100}{27}$$

$$\Rightarrow k = \pm 300$$

26. Two circles with equal radii are intersecting at the points $(0, 1)$ and $(0, -1)$. The tangent at the point $(0, 1)$ to one of the circles passes through the centre of the other circle. Then the distance between the centres of these circles is :

- (1) 1 (2) $\sqrt{2}$
(3) $2\sqrt{2}$ (4) 2

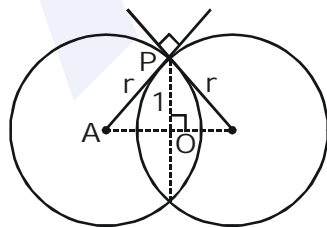
Ans. (4)

Sol. In ΔAPO

$$\left(\frac{\sqrt{2}r}{2}\right)^2 + 1^2 = r^2$$

$$\Rightarrow \boxed{r = \sqrt{2}}$$

So distance between centres $= \sqrt{2}r = 2$



27. Equation of a common tangent to the parabola $y^2 = 4x$ and the hyperbola $xy = 2$ is :

- (1) $x + 2y + 4 = 0$
(2) $x - 2y + 4 = 0$
(3) $x + y + 1 = 0$
(4) $4x + 2y + 1 = 0$

Ans. (1)

Sol. Let the equation of tangent to parabola

$$y^2 = 4x \text{ be } y = mx + \frac{1}{m}$$

It is also a tangent to hyperbola $xy = 2$

$$\Rightarrow x\left(mx + \frac{1}{m}\right) = 2$$

$$\Rightarrow x^2m + \frac{x}{m} - 2 = 0$$

$$D = 0 \Rightarrow m = -\frac{1}{2}$$

So tangent is $2y + x + 4 = 0$

28. The plane containing the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-1}{3}$

and also containing its projection on the plane $2x + 3y - z = 5$, contains which one of the following points ?

- (1) $(2, 0, -2)$ (2) $(-2, 2, 2)$
(3) $(0, -2, 2)$ (4) $(2, 2, 0)$

Ans. (1)

Sol. The normal vector of required plane

$$= (2\hat{i} - \hat{j} + 3\hat{k}) \times (2\hat{i} + 3\hat{j} - \hat{k})$$

$$= -8\hat{i} + 8\hat{j} + 8\hat{k}$$

So, direction ratio of normal is $(-1, 1, 1)$

So required plane is

$$-(x-3) + (y+2) + (z-1) = 0$$

$$\Rightarrow -x + y + z + 4 = 0$$

Which is satisfied by $(2, 0, -2)$

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29. If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$ at all points on the ellipse other than its four vertices then the mid points of the tangents intercepted between the coordinate axes lie on the curve :

- (1) $\frac{x^2}{2} + \frac{y^2}{4} = 1$ (2) $\frac{x^2}{4} + \frac{y^2}{2} = 1$
 (3) $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$ (4) $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$

Ans. (3)

Sol. Equation of general tangent on ellipse

$$\frac{x}{a \sec \theta} + \frac{y}{b \operatorname{cosec} \theta} = 1$$

$$a = \sqrt{2}, b = 1$$

$$\Rightarrow \frac{x}{\sqrt{2} \sec \theta} + \frac{y}{\operatorname{cosec} \theta} = 1$$

Let the midpoint be (h, k)

$$h = \frac{\sqrt{2} \sec \theta}{2} \Rightarrow \cos \theta = \frac{1}{\sqrt{2}h}$$

$$\text{and } k = \frac{\operatorname{cosec} \theta}{2} \Rightarrow \sin \theta = \frac{1}{2k}$$

$$\therefore \sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow \frac{1}{2h^2} + \frac{1}{4k^2} = 1$$

$$\Rightarrow \frac{1}{2x^2} + \frac{1}{4y^2} = 1$$

30. Two integers are selected at random from the set $\{1, 2, \dots, 11\}$. Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is :

- (1) $\frac{2}{5}$
 (2) $\frac{1}{2}$
 (3) $\frac{3}{5}$
 (4) $\frac{7}{10}$

Ans. (1)

Sol. Since sum of two numbers is even so either both are odd or both are even. Hence number of elements in reduced samples space
 $= {}^5C_2 + {}^6C_2$

$$\text{so required probability} = \frac{{}^5C_2}{{}^5C_2 + {}^6C_2}$$

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