

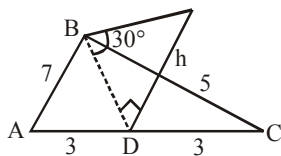
**TEST PAPER OF JEE(MAIN) EXAMINATION – 2019**  
**(Held On Thursday 10<sup>th</sup> JANUARY, 2019) TIME : 9 : 30 AM To 12 : 30 PM**  
**MATHEMATICS**

1. Consider a triangular plot ABC with sides AB=7m, BC=5m and CA=6m. A vertical lamp-post at the mid point D of AC subtends an angle 30° at B. The height (in m) of the lamp-post is:

- (1)  $7\sqrt{3}$     (2)  $\frac{2}{3}\sqrt{21}$     (3)  $\frac{3}{2}\sqrt{21}$     (4)  $2\sqrt{21}$

Ans. (2)

Sol.



$$BD = h \cot 30^\circ = h\sqrt{3}$$

$$\text{So, } 7^2 + 5^2 = 2(h\sqrt{3})^2 + 3^2$$

$$\Rightarrow 37 = 3h^2 + 9.$$

$$\Rightarrow 3h^2 = 28$$

$$\Rightarrow h = \sqrt{\frac{28}{3}} = \frac{2}{3}\sqrt{21}$$

2. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function such that  $f(x) = x^3 + x^2f'(1) + xf''(2) + f'''(3)$ ,  $x \in \mathbb{R}$ .

Then  $f(2)$  equal :

- (1) 8    (2) -2    (3) -4    (4) 30

Ans. (2)

Sol.  $f(x) = x^3 + x^2f'(1) + xf''(2) + f'''(3)$

$$\Rightarrow f'(x) = 3x^2 + 2xf'(1) + f''(x) \quad \dots(1)$$

$$\Rightarrow f''(x) = 6x + 2f'(1) \quad \dots(2)$$

$$\Rightarrow f'''(x) = 6 \quad \dots(3)$$

put  $x = 1$  in equation (1) :

$$f'(1) = 3 + 2f'(1) + f''(2) \quad \dots(4)$$

put  $x = 2$  in equation (2) :

$$f''(2) = 12 + 2f'(1) \quad \dots(5)$$

from equation (4) & (5) :

$$-3 - f'(1) = 12 + 2f'(1)$$

$$\Rightarrow 3f'(1) = -15$$

$$\Rightarrow f'(1) = -5 \Rightarrow f''(2) = 2 \quad \dots(2)$$

put  $x = 3$  in equation (3) :

$$f'''(3) = 6$$

$$\therefore f(x) = x^3 - 5x^2 + 2x + 6$$

$$f(2) = 8 - 20 + 4 + 6 = -2$$

3. If a circle C passing through the point (4,0) touches the circle  $x^2 + y^2 + 4x - 6y = 12$  externally at the point (1, -1), then the radius of C is :

- (1)  $\sqrt{57}$     (2) 4    (3)  $2\sqrt{5}$     (4) 5

Ans. (4)

Sol.  $x^2 + y^2 + 4x - 6y - 12 = 0$

Equation of tangent at (1, -1)

$$x - y + 2(x + 1) - 3(y - 1) - 12 = 0$$

$$3x - 4y - 7 = 0$$

$\therefore$  Equation of circle is

$$(x^2 + y^2 + 4x - 6y - 12) + \lambda(3x - 4y - 7) = 0$$

It passes through (4, 0) :

$$(16 + 16 - 12) + \lambda(12 - 7) = 0$$

$$\Rightarrow 20 + \lambda(5) = 0$$

$$\Rightarrow \lambda = -4$$

$$\therefore (x^2 + y^2 + 4x - 6y - 12) - 4(3x - 4y - 7) = 0$$

$$\text{or } x^2 + y^2 - 8x + 10y + 16 = 0$$

$$\text{Radius} = \sqrt{16 + 25 - 16} = 5$$

4. In a class of 140 students numbered 1 to 140, all even numbered students opted mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is :

- (1) 102    (2) 42    (3) 1    (4) 38

Ans. (4)

Sol. Let  $n(A)$  = number of students opted Mathematics = 70,

$n(B)$  = number of students opted Physics = 46,

$n(C)$  = number of students opted Chemistry = 28,

$$n(A \cap B) = 23,$$

$n(B \cap C) = 9,$   
 $n(A \cap C) = 14,$   
 $n(A \cap B \cap C) = 4,$   
 Now  $n(A \cup B \cup C)$   
 $= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C)$   
 $- n(A \cap C) + n(A \cap B \cap C)$   
 $= 70 + 46 + 28 - 23 - 9 - 14 + 4 = 102$   
 So number of students not opted for any course  
 $= \text{Total} - n(A \cup B \cup C)$   
 $= 140 - 102 = 38$

5. The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is :

- (1) 1365 (2) 1256 (3) 1465 (4) 1356

Ans. (4)

Sol.  $\sum_{r=2}^{13} (7r+2) = 7 \cdot \frac{2+13}{2} \times 6 + 2 \times 12$   
 $= 7 \times 90 + 24 = 654$

$\sum_{r=1}^{13} (7r+5) = 7 \left( \frac{1+13}{2} \right) \times 13 + 5 \times 13 = 702$

Total = 654 + 702 = 1356

6. Let  $\vec{a} = 2\hat{i} + \lambda_1\hat{j} + 3\hat{k}$ ,  $\vec{b} = 4\hat{i} + (3 - \lambda_2)\hat{j} + 6\hat{k}$  and  $\vec{c} = 3\hat{i} + 6\hat{j} + (\lambda_3 - 1)\hat{k}$  be three vectors such that  $\vec{b} = 2\vec{a}$  and  $\vec{a}$  is perpendicular to  $\vec{c}$ . Then a possible value of  $(\lambda_1, \lambda_2, \lambda_3)$  is :-

- (1)  $\left(\frac{1}{2}, 4, -2\right)$  (2)  $\left(-\frac{1}{2}, 4, 0\right)$   
 (3) (1, 3, 1) (4) (1, 5, 1)

Ans. (2)

Sol.  $4\hat{i} + (3 - \lambda_2)\hat{j} + 6\hat{k} = 4\hat{i} + 2\lambda_1\hat{j} + 6\hat{k}$   
 $\Rightarrow 3 - \lambda_2 = 2\lambda_1 \Rightarrow 2\lambda_1 + \lambda_2 = 3 \dots(1)$

Given  $\vec{a} \cdot \vec{c} = 0$   
 $\Rightarrow 6 + 6\lambda_1 + 3(\lambda_3 - 1) = 0$   
 $\Rightarrow 2\lambda_1 + \lambda_3 = -1 \dots(2)$

Now  $(\lambda_1, \lambda_2, \lambda_3) = (\lambda_1, 3 - 2\lambda_1, -1 - 2\lambda_1)$

Now check the options, option (2) is correct

7. The equation of a tangent to the hyperbola  $4x^2 - 5y^2 = 20$  parallel to the line  $x - y = 2$  is :

- (1)  $x - y + 9 = 0$   
 (2)  $x - y + 7 = 0$   
 (3)  $x - y + 1 = 0$   
 (4)  $x - y - 3 = 0$

Ans. (3)

Sol. Hyperbola  $\frac{x^2}{5} - \frac{y^2}{4} = 1$

slope of tangent = 1

equation of tangent  $y = x \pm \sqrt{5-4}$

$\Rightarrow y = x \pm 1$

$\Rightarrow y = x + 1$  or  $y = x - 1$

8. If the area enclosed between the curves  $y=kx^2$  and  $x=ky^2$ , ( $k>0$ ), is 1 square unit. Then k is:

- (1)  $\frac{1}{\sqrt{3}}$  (2)  $\frac{2}{\sqrt{3}}$  (3)  $\frac{\sqrt{3}}{2}$  (4)  $\sqrt{3}$

Ans. (1)

Sol. Area bounded by  $y^2 = 4ax$  &  $x^2 = 4by$ ,  $a, b \neq 0$

is  $\left| \frac{16ab}{3} \right|$

by using formula :  $4a = \frac{1}{k} = 4b, k > 0$

Area =  $\left| \frac{16 \cdot \frac{1}{4k} \cdot \frac{1}{4k}}{3} \right| = 1$

$\Rightarrow k^2 = \frac{1}{3}$

$\Rightarrow k = \frac{1}{\sqrt{3}}$

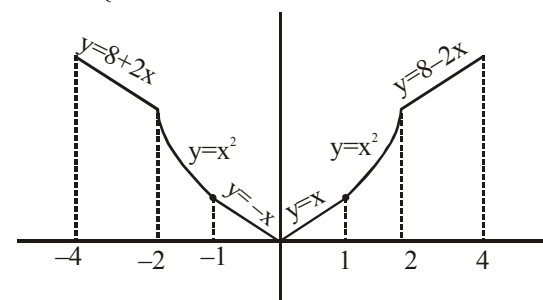
9. Let  $f(x) = \begin{cases} \max\{|x|, x^2\}, & |x| \leq 2 \\ 8 - 2|x|, & 2 < |x| \leq 4 \end{cases}$

Let S be the set of points in the interval  $(-4, 4)$  at which f is not differentiable. Then S:

- (1) is an empty set  
 (2) equals  $\{-2, -1, 1, 2\}$   
 (3) equals  $\{-2, -1, 0, 1, 2\}$   
 (4) equals  $\{-2, 2\}$

Ans. (3)

Sol.  $f(x) = \begin{cases} 8 + 2x, & -4 \leq x < -2 \\ x^2, & -2 \leq x \leq -1 \\ |x|, & -1 < x < 1 \\ x^2, & 1 \leq x \leq 2 \\ 8 - 2x, & 2 < x \leq 4 \end{cases}$



$f(x)$  is not differentiable at  $x = \{-2, -1, 0, 1, 2\}$

$\Rightarrow S = \{-2, -1, 0, 1, 2\}$

10. If the parabolas  $y^2=4b(x-c)$  and  $y^2=8ax$  have a common normal, then which one of the following is a valid choice for the ordered triad (a,b,c)

- (1) (1, 1, 0)                      (2)  $\left(\frac{1}{2}, 2, 3\right)$   
(3)  $\left(\frac{1}{2}, 2, 0\right)$                       (4) (1, 1, 3)

**Ans. (1,2,3,4)**

**Sol.** Normal to these two curves are

$$y = m(x - c) - 2bm - bm^3,$$

$$y = mx - 4am - 2am^3$$

If they have a common normal  
(c + 2b) m + bm<sup>3</sup> = 4am + 2am<sup>3</sup>

$$\text{Now } (4a - c - 2b) m = (b - 2a)m^3$$

We get all options are correct for m = 0  
(common normal x-axis)

Ans. (1), (2), (3), (4)

**Remark :**

If we consider question as

If the parabolas  $y^2 = 4b(x - c)$  and  $y^2 = 8ax$  have a common normal other than x-axis, then which one of the following is a valid choice for the ordered triad (a, b, c) ?

$$\text{When } m \neq 0 : (4a - c - 2b) = (b - 2a)m^2$$

$$m^2 = \frac{c}{2a - b} - 2 > 0 \Rightarrow \frac{c}{2a - b} > 2$$

Now according to options, option 4 is correct

11. The sum of all values of  $\theta \in \left(0, \frac{\pi}{2}\right)$  satisfying

$$\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4} \text{ is :}$$

- (1)  $\frac{\pi}{2}$                       (2)  $\pi$                       (3)  $\frac{3\pi}{8}$                       (4)  $\frac{5\pi}{4}$

**Ans. (1)**

**Sol.**  $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$ ,  $\theta \in \left(0, \frac{\pi}{2}\right)$

$$\Rightarrow 1 - \cos^2 2\theta + \cos^4 2\theta = \frac{3}{4}$$

$$\Rightarrow 4\cos^4 2\theta - 4\cos^2 2\theta + 1 = 0$$

$$\Rightarrow (2\cos^2 2\theta - 1)^2 = 0$$

$$\Rightarrow \cos^2 2\theta = \frac{1}{2} = \cos^2 \frac{\pi}{4}$$

$$\Rightarrow 2\theta = n\pi \pm \frac{\pi}{4}, n \in I$$

$$\Rightarrow \theta = \frac{n\pi}{2} \pm \frac{\pi}{8}$$

$$\Rightarrow \theta = \frac{\pi}{8}, \frac{\pi}{2}, \frac{\pi}{8}$$

Sum of solutions  $\frac{\pi}{2}$

12. Let  $z_1$  and  $z_2$  be any two non-zero complex numbers such that  $3|z_1| = 4|z_2|$ .

If  $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$  then :

(1)  $|z| = \frac{1}{2}\sqrt{\frac{17}{2}}$                       (2)  $\text{Re}(z) = 0$

(3)  $|z| = \sqrt{\frac{5}{2}}$                       (4)  $\text{Im}(z) = 0$

**Ans. (Bonus)**

**Sol.**  $3|z_1| = 4|z_2|$

$$\Rightarrow \frac{|z_1|}{|z_2|} = \frac{4}{3}$$

$$\Rightarrow \frac{|3z_1|}{|2z_2|} = 2$$

$$\text{Let } \frac{3z_1}{2z_2} = a = 2\cos\theta + 2i\sin\theta$$

$$z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1} = a + \frac{1}{a}$$

$$= \frac{5}{2}\cos\theta + \frac{3}{2}i\sin\theta$$

Now all options are incorrect

**Remark :**

There is a misprint in the problem actual problem should be :

"Let  $z_1$  and  $z_2$  be any non-zero complex number such that  $3|z_1| = 2|z_2|$ ."

If  $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$ , then"

Given

$$3|z_1| = 2|z_2|$$

$$\text{Now } \left|\frac{3z_1}{2z_2}\right| = 1$$

Let  $\frac{3z_1}{2z_2} = a = \cos\theta + i\sin\theta$

$$z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$$

$$= a + \frac{1}{a} = 2\cos\theta$$

$\therefore \text{Im}(z) = 0$

Now option (4) is correct.

13. If the system of equations

$$x+y+z = 5$$

$$x+2y+3z = 9$$

$$x+3y+\alpha z = \beta$$

has infinitely many solutions, then  $\beta-\alpha$  equals:

- (1) 5      (2) 18      (3) 21      (4) 8

Ans. (4)

Sol.  $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \alpha \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & \alpha-1 \end{vmatrix} = (\alpha-1) - 4 = (\alpha-5)$

for infinite solutions  $D = 0 \Rightarrow \alpha = 5$

$$D_x = 0 \Rightarrow \begin{vmatrix} 5 & 1 & 1 \\ 9 & 2 & 3 \\ \beta & 3 & 5 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ -1 & -1 & 3 \\ \beta-15 & -2 & 5 \end{vmatrix} = 0$$

$$\Rightarrow 2 + \beta - 15 = 0 \Rightarrow \beta - 13 = 0$$

on  $\beta = 13$  we get  $D_y = D_z = 0$

$$\alpha = 5, \beta = 13$$

14. The shortest distance between the point  $\left(\frac{3}{2}, 0\right)$

and the curve  $y = \sqrt{x}, (x > 0)$  is :

- (1)  $\frac{\sqrt{5}}{2}$       (2)  $\frac{5}{4}$       (3)  $\frac{3}{2}$       (4)  $\frac{\sqrt{3}}{2}$

Ans. (1)

Sol. Let points  $\left(\frac{3}{2}, 0\right), (t^2, t), t > 0$

$$\text{Distance} = \sqrt{t^2 + \left(t^2 - \frac{3}{2}\right)^2}$$

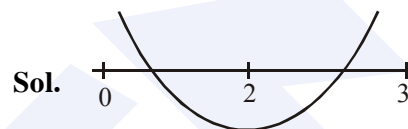
$$= \sqrt{t^4 - 2t^2 + \frac{9}{4}} = \sqrt{(t^2 - 1)^2 + \frac{5}{4}}$$

So minimum distance is  $\sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$

15. Consider the quadratic equation  $(c-5)x^2 - 2cx + (c-4) = 0, c \neq 5$ . Let S be the set of all integral values of c for which one root of the equation lies in the interval (0,2) and its other root lies in the interval (2,3). Then the number of elements in S is :

- (1) 11      (2) 18      (3) 10      (4) 12

Ans. (1)



Sol.

Let  $f(x) = (c-5)x^2 - 2cx + c-4$

$$\therefore f(0)f(2) < 0 \quad \dots(1)$$

$$\& f(2)f(3) < 0 \quad \dots(2)$$

from (1) & (2)

$$(c-4)(c-24) < 0$$

$$\& (c-24)(4c-49) < 0$$

$$\Rightarrow \frac{49}{4} < c < 24$$

$$\therefore s = \{13, 14, 15, \dots, 23\}$$

Number of elements in set S = 11

16.  $\sum_{i=1}^{20} \left( \frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}} \right) = \frac{k}{21}$ , then k equals :

- (1) 200      (2) 50      (3) 100      (4) 400

Ans. (3)

Sol.  $\sum_{i=1}^{20} \left( \frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}} \right)^3 = \frac{k}{21}$

$$\Rightarrow \sum_{i=1}^{20} \left( \frac{{}^{20}C_{i-1}}{{}^{21}C_i} \right)^3 = \frac{k}{21}$$

$$\Rightarrow \sum_{i=1}^{20} \left( \frac{i}{21} \right)^3 = \frac{k}{21}$$

$$\Rightarrow \frac{1}{(21)^3} \left[ \frac{20(21)}{2} \right]^2 = \frac{k}{21}$$

$$\Rightarrow 100 = k$$



21. A point P moves on the line  $2x - 3y + 4 = 0$ . If Q(1,4) and R(3,-2) are fixed points, then the locus of the centroid of  $\Delta PQR$  is a line :

- (1) parallel to x-axis      (2) with slope  $\frac{2}{3}$   
 (3) with slope  $\frac{3}{2}$       (4) parallel to y-axis

Ans. (2)

Sol. Let the centroid of  $\Delta PQR$  is (h, k) & P is  $(\alpha, \beta)$ , then

$$\frac{\alpha + 1 + 3}{3} = h \quad \text{and} \quad \frac{\beta + 4 - 2}{3} = k$$

$$\alpha = (3h - 4) \quad \beta = (3k - 4)$$

Point P( $\alpha, \beta$ ) lies on line  $2x - 3y + 4 = 0$   
 $\therefore 2(3h - 4) - 3(3k - 2) + 4 = 0$   
 $\Rightarrow$  locus is  $6x - 9y + 2 = 0$

22. If  $\frac{dy}{dx} + \frac{3}{\cos^2 x} y = \frac{1}{\cos^2 x}$ ,  $x \in \left(\frac{-\pi}{3}, \frac{\pi}{3}\right)$ , and

$$y\left(\frac{\pi}{4}\right) = \frac{4}{3}, \text{ then } y\left(-\frac{\pi}{4}\right) \text{ equals :}$$

- (1)  $\frac{1}{3} + e^6$       (2)  $\frac{1}{3}$   
 (3)  $-\frac{4}{3}$       (4)  $\frac{1}{3} + e^3$

Ans. (1)

Sol.  $\frac{dy}{dx} + 3\sec^2 x \cdot y = \sec^2 x$

$$\text{I.F.} = e^{3\int \sec^2 x dx} = e^{3\tan x}$$

$$\text{or } y \cdot e^{3\tan x} = \int \sec^2 x \cdot e^{3\tan x} dx$$

$$\text{or } y \cdot e^{3\tan x} = \frac{1}{3} e^{3\tan x} + C \quad \dots(1)$$

Given

$$y\left(\frac{\pi}{4}\right) = \frac{4}{3}$$

$$\therefore \frac{4}{3} \cdot e^3 = \frac{1}{3} e^3 + C$$

$$\therefore C = e^3$$

Now put  $x = -\frac{\pi}{4}$  in equation (1)

$$\therefore y \cdot e^{-3} = \frac{1}{3} e^{-3} + e^3$$

$$\therefore y = \frac{1}{3} + e^6$$

$$\therefore y\left(-\frac{\pi}{4}\right) = \frac{1}{3} + e^6$$

23. The plane passing through the point (4, -1, 2)

and parallel to the lines  $\frac{x+2}{3} = \frac{y-2}{-1} = \frac{z+1}{2}$

and  $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{3}$  also passes through the point :

- (1) (-1, -1, -1)      (2) (-1, -1, 1)  
 (3) (1, 1, -1)      (4) (1, 1, 1)

Ans. (4)

Sol. Let  $\vec{n}$  be the normal vector to the plane passing through (4, -1, 2) and parallel to the lines  $L_1$  &  $L_2$

$$\text{then } \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$$

$$\therefore \vec{n} = -7\hat{i} - 7\hat{j} + 7\hat{k}$$

$\therefore$  Equation of plane is

$$-1(x - 4) - 1(y + 1) + 1(z - 2) = 0$$

$$\therefore x + y - z - 1 = 0$$

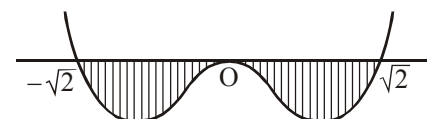
Now check options

24. Let  $I = \int_a^b (x^4 - 2x^2) dx$ . If I is minimum then the ordered pair (a, b) is :

- (1)  $(-\sqrt{2}, 0)$       (2)  $(-\sqrt{2}, \sqrt{2})$   
 (3)  $(0, \sqrt{2})$       (4)  $(\sqrt{2}, -\sqrt{2})$

Ans. (2)

Sol. Let  $f(x) = x^2(x^2 - 2)$



As long as  $f(x)$  lie below the x-axis, definite integral will remain negative,

so correct value of (a, b) is  $(-\sqrt{2}, \sqrt{2})$  for minimum of I

25. If 5, 5r, 5r<sup>2</sup> are the lengths of the sides of a triangle, then r cannot be equal to :

- (1)  $\frac{3}{2}$       (2)  $\frac{3}{4}$       (3)  $\frac{5}{4}$       (4)  $\frac{7}{4}$

**Ans. (4)**

**Sol.** r = 1 is obviously true.

Let 0 < r < 1

$$\Rightarrow r + r^2 > 1$$

$$\Rightarrow r^2 + r - 1 > 0$$

$$\left( r - \frac{-1 - \sqrt{5}}{2} \right) \left( r - \frac{-1 + \sqrt{5}}{2} \right)$$

$$\Rightarrow r - \frac{-1 - \sqrt{5}}{2} \text{ or } r > \frac{-1 + \sqrt{5}}{2}$$

$$r \in \left( \frac{\sqrt{5} - 1}{2}, 1 \right)$$

$$\frac{\sqrt{5} - 1}{2} < r < 1$$

When r > 1

$$\Rightarrow \frac{\sqrt{5} + 1}{2} > \frac{1}{r} > 1$$

$$\Rightarrow r \in \left( \frac{\sqrt{5} - 1}{2}, \frac{\sqrt{5} + 1}{2} \right)$$

Now check options

26. Consider the statement : "P(n): n<sup>2</sup> - n + 41 is prime." Then which one of the following is true?

- (1) P(5) is false but P(3) is true  
 (2) Both P(3) and P(5) are false  
 (3) P(3) is false but P(5) is true  
 (4) Both P(3) and P(5) are true

**Ans. (4)**

**Sol.** P(n) : n<sup>2</sup> - n + 41 is prime

P(5) = 61 which is prime

P(3) = 47 which is also prime

27. Let A be a point on the line

$$\vec{r} = (1 - 3\mu)\hat{i} + (\mu - 1)\hat{j} + (2 + 5\mu)\hat{k} \text{ and } B(3, 2, 6)$$

be a point in the space. Then the value of  $\mu$  for which the vector  $\overline{AB}$  is parallel to the plane

$$x - 4y + 3z = 1 \text{ is :}$$

- (1)  $\frac{1}{2}$       (2)  $-\frac{1}{4}$       (3)  $\frac{1}{4}$       (4)  $\frac{1}{8}$

**Ans. (3)**

**Sol.** Let point A is

$$(1 - 3\mu)\hat{i} + (\mu - 1)\hat{j} + (2 + 5\mu)\hat{k}$$

and point B is (3, 2, 6)

$$\text{then } \overline{AB} = (2 + 3\mu)\hat{i} + (3 - \mu)\hat{j} + (4 - 5\mu)\hat{k}$$

which is parallel to the plane  $x - 4y + 3z = 1$

$$\therefore 2 + 3\mu - 12 + 4\mu + 12 - 15\mu = 0$$

$$8\mu = 2$$

$$\mu = \frac{1}{4}$$

28. For each  $t \in \mathbb{R}$ , let  $[t]$  be the greatest integer less than or equal to t. Then,

$$\lim_{x \rightarrow 1^+} \frac{(1 - |x| + \sin |1 - x|) \sin \left( \frac{\pi}{2} [1 - x] \right)}{|1 - x| [1 - x]}$$

- (1) equals -1      (2) equals 1  
 (3) does not exist      (4) equals 0

**Ans. (4)**

$$\text{Sol. } \lim_{x \rightarrow 1^+} \frac{(1 - |x| + \sin |1 - x|) \sin \left( \frac{\pi}{2} [1 - x] \right)}{|1 - x| [1 - x]}$$

$$= \lim_{x \rightarrow 1^+} \frac{(1 - x) + \sin(x - 1)}{(x - 1)(-1)} \sin \left( \frac{\pi}{2} (-1) \right)$$

$$= \lim_{x \rightarrow 1^+} \left( 1 - \frac{\sin(x - 1)}{(x - 1)} \right) (-1) = (1 - 1)(-1) = 0$$

**MAJOR COMPUTER BASED TEST (CBT) SERIES**

**JEE (Advanced)- Target 2019**

[dlp.allen.ac.in](http://dlp.allen.ac.in)

Test Dates: 3<sup>rd</sup> Feb, 21<sup>st</sup> & 28<sup>th</sup> April, 12<sup>th</sup> May

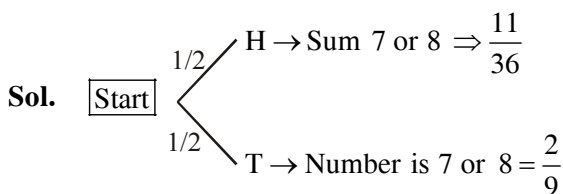
0744-2750275



29. An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rolled and the sum of the numbers obtained on them is noted. If the toss of the coin results in tail then a card from a well-shuffled pack of nine cards numbered 1,2,3,...,9 is randomly picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is :

- (1)  $\frac{13}{36}$       (2)  $\frac{19}{36}$       (3)  $\frac{19}{72}$       (4)  $\frac{15}{72}$

Ans. (3)



$$P(A) = \frac{1}{2} \times \frac{11}{36} + \frac{1}{2} \times \frac{2}{9} = \frac{19}{72}$$

30. Let  $n \geq 2$  be a natural number and  $0 < \theta < \pi/2$ .

Then  $\int \frac{(\sin^n \theta - \sin \theta) \cos \theta}{\sin^{n+1} \theta} d\theta$  is equal to :

(Where C is a constant of integration)

(1)  $\frac{n}{n^2-1} \left(1 - \frac{1}{\sin^{n+1} \theta}\right)^{\frac{n+1}{n}} + C$

(2)  $\frac{n}{n^2+1} \left(1 - \frac{1}{\sin^{n-1} \theta}\right)^{\frac{n+1}{n}} + C$

(3)  $\frac{n}{n^2-1} \left(1 - \frac{1}{\sin^{n-1} \theta}\right)^{\frac{n+1}{n}} + C$

(4)  $\frac{n}{n^2-1} \left(1 + \frac{1}{\sin^{n-1} \theta}\right)^{\frac{n+1}{n}} + C$

Ans. (3)

Sol. 
$$\int \frac{(\sin^n \theta - \sin \theta)^{1/n} \cos \theta}{\sin^{n+1} \theta} d\theta$$

$$= \int \frac{\sin \theta \left(1 - \frac{1}{\sin^{n-1} \theta}\right)^{1/n}}{\sin^{n+1} \theta} d\theta$$

Put  $1 - \frac{1}{\sin^{n-1} \theta} = t$

So  $\frac{(n-1)}{\sin^n \theta} \cos \theta d\theta = dt$

Now  $\frac{1}{n-1} \int (t)^{1/n} dt$

$$= \frac{1}{(n-1)} \frac{(t)^{\frac{1}{n}+1}}{\frac{1}{n}+1} + C$$

$$= \frac{1}{(n-1)} \left(1 - \frac{1}{\sin^{n-1} \theta}\right)^{\frac{1}{n}+1} + C$$

**MAJOR COMPUTER BASED TEST (CBT) SERIES**

**JEE (Main)- Target 2019**

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Test Dates: 24<sup>th</sup> & 31<sup>st</sup> March

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