TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Thursday 10th JANUARY, 2019) TIME : 9 : 30 AM To 12 : 30 PM

MATHEMATICS

1. Consider a triangular plot ABC with sides AB=7m, BC=5m and CA=6m. A vertical lamp-post at the mid point D of AC subtends an angle 30° at B. The height (in m) of the lamp-post is:

(1) $7\sqrt{3}$ (2) $\frac{2}{3}\sqrt{21}$ (3) $\frac{3}{2}\sqrt{21}$ (4) $2\sqrt{21}$

Ans. (2)

2. Let f : R→R be a function such that $f(x) = x^3+x^2f'(1) + xf''(2)+f'''(3)$, $x\in R$.
Then f(2) equal :
(1) 8 (2) –2 (3) –4 (4) 30

Ans. (2)

3. If a circle C passing through the point (4,0) touches the circle $x^2 + y^2 + 4x - 6y = 12$ externally at the point (1, –1), then the radius of C is :

(1) $\sqrt{57}$ (2) 4 (3) $2\sqrt{5}$ (4) 5

Ans. (4)

4. In a class of 140 students numbered 1 to 140, all even numbered students opted mathematics course, those whose number is divisible by 3 opted Physics course and theose whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is :
(1) 102 (2) 42 (3) 1 (4) 38

Ans. (4)

5. The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is :
(1) 1365 (2) 1256 (3) 1465 (4) 1356

Ans. (4)

6. Let $\vec{a} = 2\hat{i}+\lambda_1\hat{j}+3\hat{k}$, $\vec{b} = 4\hat{i}+(3-\lambda_2)\hat{j}+6\hat{k}$ and $\vec{c} = 3\hat{i}+6\hat{j}+(\lambda_3-1)\hat{k}$ be three vectors such that $\vec{b} = 2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$. Then a possible value of $(\lambda_1,\lambda_2,\lambda_3)$ is :-
(1) $\left(\frac{1}{2},4,-2\right)$ (2) $\left(-\frac{1}{2},4,0\right)$
(3) (1,3,1) (4) (1,5,1)

Ans. (2)

7. The equation of a tangent to the hyperbola $4x^2-5y^2 = 20$ parallel to the line x–y = 2 is :
(1) x–y+9 = 0
(2) x–y+7 = 0
(3) x–y+1 = 0
(4) x–y–3 = 0

Ans. (3)

8. If the area enclosed between the curves $y=kx^2$ and $x=ky^2$, (k>0), is 1 square unit. Then k is:

(1) $\frac{1}{\sqrt{3}}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\sqrt{3}$

Ans. (1)

9. Let $f(x)=\begin{cases}\max\{|x|,x^2\}, & |x|\le 2\\ 8-2|x|, & 2<|x|\le 4\end{cases}$

Let S be the set of points in the interval (–4,4) at which f is not differentiable. Then S:
(1) is an empty set
(2) equals {–2, –1, 1, 2}
(3) equals {–2, –1, 0, 1, 2}
(4) equals {–2, 2}

Ans. (3)

10. If the parabolas $y^2=4b(x-c)$ and $y^2=8ax$ have a common normal, then which one of the following is a valid choice for the ordered triad (a,b,c)

(1) (1, 1, 0) (2) $\left(\frac{1}{2},2,3\right)$

(3) $\left(\frac{1}{2},2,0\right)$ (4) (1, 1, 3)

Ans. (1,2,3,4)

11. The sum of all values of $\theta \in \left(0, \frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$ is :

(1) $\frac{\pi}{2}$ (2) π (3) $\frac{3\pi}{8}$ (4) $\frac{5\pi}{4}$

Ans. (1)

12. Let z_1 and z_2 be any two non-zero complex numbers such that $3|z_1| = 4|z_2|$.

If $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$ then :

(1) $|z| = \frac{1}{2}\sqrt{\frac{17}{2}}$ (2) Re(z) = 0

(3) $|z| = \sqrt{\frac{5}{2}}$ (4) Im(z) = 0

Ans. (Bonus)

13. If the system of equations

$x+y+z = 5$

$x+2y+3z = 9$

$x+3y+\alpha z = \beta$

has infinitely many solutions, then $\beta-\alpha$ equals:

(1) 5 (2) 18 (3) 21 (4) 8

Ans. (4)

14. The shortest distance between the point $\left(\frac{3}{2}, 0\right)$ and the curve $y = \sqrt{x}, (x > 0)$ is :

(1) $\frac{\sqrt{5}}{2}$ (2) $\frac{5}{4}$ (3) $\frac{3}{2}$ (4) $\frac{\sqrt{3}}{2}$

Ans. (1)

15. Consider the quadratic equation $(c-5)x^2-2cx + (c-4) = 0$, $c \neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0,2) and its other root lies in the interval (2,3). Then the number of elements in S is :

(1) 11 (2) 18 (3) 10 (4) 12

Ans. (1)

16. $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right) = \frac{k}{21}$, then k equals :

(1) 200 (2) 50 (3) 100 (4) 400

Ans. (3)

17. Let $d \in R$, and

$$A = \begin{bmatrix} -2 & 4+d & (\sin\theta)-2 \\ 1 & (\sin\theta)+2 & d \\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d \end{bmatrix},$$

$\theta \in [0, 2\pi]$. If the minimum value of det(A) is 8, then a value of d is :

(1) –7 (2) $2\left(\sqrt{2}+2\right)$

(3) –5 (4) $2\left(\sqrt{2}+1\right)$

Ans. (3)

18. If the third term in the binomial expansion of $\left(1 + x^{\log_2 x}\right)^5$ equals 2560, then a possible value of x is :

(1) $2\sqrt{2}$ (2) $\frac{1}{8}$ (3) $4\sqrt{2}$ (4) $\frac{1}{4}$

Ans. (4)

19. If the line $3x + 4y - 24 = 0$ intersects the x-axis at the point A and the y-axis at the point B, then the incentre of the triangle OAB, where O is the origin, is

(1) (3, 4) (2) (2, 2) (3) (4, 4) (4) (4, 3)

Ans. (2)

20. The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is :

(1) 4 : 9 (2) 6 : 7

(3) 5 : 8 (4) 10 : 3

Ans. (1)

21. A point P moves on the line $2x - 3y + 4 = 0$. IfQ(1,4) and R(3,–2) are fixed points, then the locus of the centroid of ΔPQR is a line :

(1) parallel to x-axis (2) with slope $\frac{2}{3}$

(3) with slope $\frac{3}{2}$ (4) parallel to y-axis

Ans. (2)

22. If $\frac{dy}{dx}+\frac{3}{\cos^2 x}y=\frac{1}{\cos^2 x}, x\in\left(\frac{-\pi}{3},\frac{\pi}{3}\right)$, and $y\left(\frac{\pi}{4}\right)=\frac{4}{3}$, then $y\left(-\frac{\pi}{4}\right)$ equals :

(1) $\frac{1}{3}+e^6$ (2) $\frac{1}{3}$

(3) $-\frac{4}{3}$ (4) $\frac{1}{3}+e^3$

Ans. (1)

23. The plane passing through the point (4, –1, 2) and parallel to the lines $\frac{x+2}{3}=\frac{y-2}{-1}=\frac{z+1}{2}$ and $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z-4}{3}$ also passes through the point :

(1) (–1, –1, –1) (2) (–1, –1, 1)

(3) (1, 1, –1) (4) (1, 1, 1)

Ans. (4)

24. Let $I=\int_a^b (x^4-2x^2)dx$. If I is minimum then the ordered pair (a, b) is :

(1) $\left(-\sqrt{2},0\right)$ (2) $\left(-\sqrt{2},\sqrt{2}\right)$

(3) $\left(0,\sqrt{2}\right)$ (4) $\left(\sqrt{2},-\sqrt{2}\right)$

Ans. (2)

25. If 5, 5r, $5r^2$ are the lengths of the sides of a triangle, then r cannot be equal to :

(1) $\frac{3}{2}$ (2) $\frac{3}{4}$

(3) $\frac{5}{4}$ (4) $\frac{7}{4}$

Ans. (4)

26. Consider the statement : "P(n): $n^2 - n + 41$ is prime." Then which one of the following is true?

(1) P(5) is false but P(3) is true

(2) Both P(3) and P(5) are false

(3) P(3) is false but P(5) is true

(4) Both P(3) and P(5) are true

Ans. (4)

27. Let A be a point on the line $\vec{r}=(1-3\mu)\hat{i}+(\mu-1)\hat{j}+(2+5\mu)\hat{k}$ and B(3, 2, 6) be a point in the space. Then the value of μ for which the vector $\overrightarrow{AB}$ is parallel to the plane $x-4y+3z=1$ is :

(1) $\frac{1}{2}$ (2) $-\frac{1}{4}$ (3) $\frac{1}{4}$ (4) $\frac{1}{8}$

Ans. (3)

28. For each $t\in R$, let [t] be the greatest integer less than or equal to t. Then,

$$\lim_{x\to 1+}\frac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$$

(1) equals –1 (2) equals 1

(3) does not exist (4) equals 0

Ans. (4)

29. An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rolled and the sum of the numbers obtained on them is noted. If the toss of the coin results in tail then a card from a well-shuffled pack of nine cards numbered 1,2,3,...,9 is randomly picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is :

(1) $\frac{13}{36}$ (2) $\frac{19}{36}$ (3) $\frac{19}{72}$ (4) $\frac{15}{72}$

Ans. (3)

30. Let $n \geq 2$ be a natural number and $0<\theta<\pi/2$.

Then $\int \frac{(\sin^n\theta - \sin\theta)^{\frac{1}{n}}\cos\theta}{\sin^{n+1}\theta}d\theta$ is equal to :

(Where C is a constant of integration)

(1) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n+1}\theta}\right)^{\frac{n+1}{n}}+C$

(2) $\frac{n}{n^2+1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

(3) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

(4) $\frac{n}{n^2-1}\left(1+\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

Ans. (3)