CAREER POINT

JEE Advanced Exam 2019

(Paper & Solution)

Date : 27 / 05 / 2019

PAPER-2

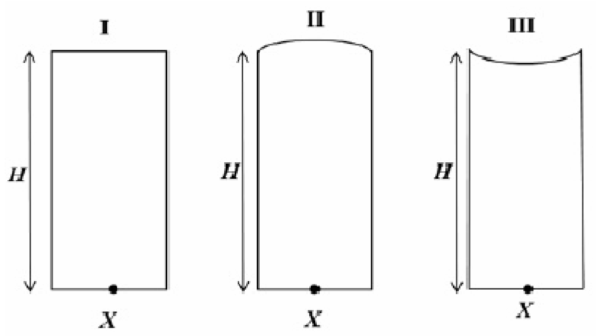
PART-I (PHYSICS)

SECTION – 1 (Maximum Marks : 32)

- This section contains **EIGHT (08)** questions
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks	: **+4**	If only (all) the correct option(s) is (are) chosen.
Partial Marks	: **+3**	If all the four options are correct but ONLY three options are chosen.
Partial Marks	: **+2**	If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks	: **+1**	If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks	: **0**	If none of the option is chosen (i.e. the question is unanswered).
Negative Marks	: **–1**	In all other cases.

Q.1 Three glass cylinders of equal height H = 30 cm and same refractive index n = 1.5 are placed on a horiozn　tal surface as shown in figure. Cylinder I has a flat top, cylinder II has a convex top and cylinder III has a concave top. The radii of curvature of the two curved tops are same (R = 3m). If H_1, H_2 and H_3 are the apparent depth of a point X on the bottom of the three cylinders, respectively, the correct statement (s) is / are –



(1) 0.8 cm < $(H_2 – H_1)$ < 0.9 cm　　(2) $H_3 > H_1$

(3) $H_2 > H_3$　　(4) $H_2 > H_1$

Ans. **[3, 4]**

Sol. **For fig. I**

$H_1 = \frac{H}{1.5} = \frac{30}{1.5} = 20$ cm

$\frac{1}{-H_2} - \frac{1.5}{-30} = \frac{1-1.5}{-300}$

For fig. II

$\frac{1}{-H_2} + \frac{1.5}{30} = \frac{1\backslash 2}{-300} = \frac{1}{600}$

$\frac{1}{H_2} = \frac{1.5}{30} - \frac{1}{600} = \frac{1}{20}\left(1 - \frac{1}{30}\right) = \frac{29}{600}$

$H_2 = 20.6$ cm.

For fig. III

$\frac{1}{-H_3} - \frac{1.5}{-30} = \frac{1-1.5}{+300}$

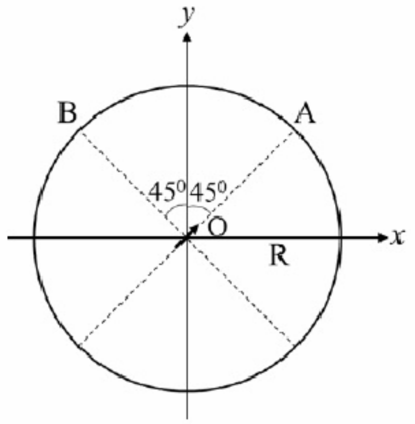
$+\frac{1}{H_2} = \frac{1}{20} + \frac{1}{600}$

$\Rightarrow H_3 = \frac{600}{31} = 19.3$

$H_2 - H_1 = 20.6 - 20 = 0.6$

$H_2 > H_1, H_3 < H_1, H_2 > H_3$

Q.2 An electric dipole with dipole moment $\frac{p_0}{\sqrt{2}}(\hat{i} + \hat{j})$ is held fixed at the origin O in the presence of an uniform electric field of magnitude E_0. If the potential is consatnt on a circle of radius R centered at the origin as shown in figure, then the correct statement (s) is/are : ($\in_0$ is permittivity of free space R > > dipole size)



(1) Total electric field at point B is $\vec{E}_B = 0$

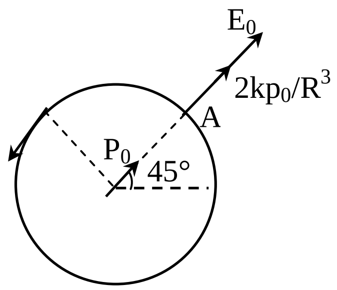
(2) Total electric field at point A is $\vec{E}_A = \sqrt{2}\,E_0(\hat{i} + \hat{j})$

(3) $R = \left(\frac{p_0}{4\pi \in_0 E_0}\right)^{1/3}$

(4) The magnitude of total electric field on any two points of the circle wil be same.

Ans. **[1,3]**

Sol.



$E_{Tangential}$ at all points on circle should be zero to make potential to be constant

$$E_A = \frac{2kp_0}{R^3} + E_0 = 2E_0 + E_0 = 3\,E_0$$

$$E_B = 0 \Rightarrow \frac{kp_0}{R^3} = E_0$$

$$E_0 = \frac{kp_0}{R^3} \Rightarrow R = \left(\frac{kp}{E_0}\right)^{\frac{1}{3}} = \left(\frac{p_0}{4\pi\in_0 E_0}\right)^{\frac{1}{3}}$$

Q.3 A thin and uniform rod of mass M and length L is held vertical on a floor with large friction. The rod is released from rest so that it falls by rotating about its contact-point with the floor without slipping. Which of the following statement (s) is/are corerct, when the rod makes an angle 60° with vertical ?

[g is the acceleration due to gravity]

(1) The angular acceleration of the rod will be $\frac{2g}{L}$

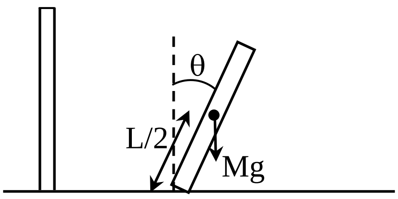
(2) The radial accelration of the rod's center of mass will be $\frac{3g}{4}$

(3) The angular speed of the rod will be $\sqrt{\frac{3g}{2L}}$

(4) The normal reaction force from the floor on the rod will be $\frac{Mg}{16}$.

Ans. **[2,3,4]**

Sol.



$$\tau = I\,\alpha \Rightarrow (Mg)\,\frac{L}{2}\sin\theta = \frac{ML^2}{3}\alpha$$

$$\alpha = \frac{3g}{2L}\sin\theta$$

At $\theta = 60°$ $\alpha = \frac{3g}{2L}\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}g}{4L}$

$a_T = \frac{L}{2}\alpha = \frac{L}{2}\frac{3\sqrt{3}g}{4L} = \frac{3\sqrt{3}g}{8}$

$Mg\left[\frac{L}{2}-\frac{L}{4}\right] = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2 \Rightarrow \frac{MgL}{4} = \frac{ML^2}{2\times3}\omega^2$

$\omega^2 = \frac{3g}{2L} \Rightarrow \omega = \sqrt{3g/2L}$

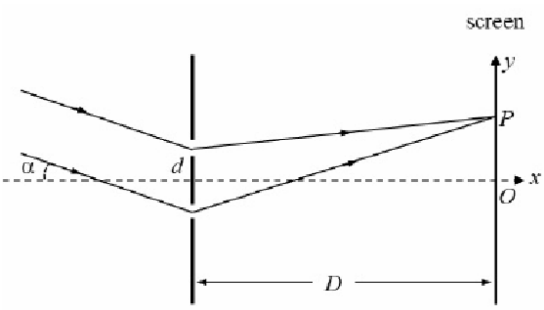
Radial acceleration $a_c = \frac{L}{2}\omega^2 = \frac{L}{2}\frac{3g}{2L} = \frac{3g}{4}$

$a_{verticaal} = \frac{3g}{4}\cos 60 + \frac{3\sqrt{3}g}{8}\sin 60° = \frac{3g}{8} + \frac{3\sqrt{3}g}{8}\frac{\sqrt{3}}{2}$

$= \frac{3g}{8}\left[1+\frac{3}{2}\right] = \frac{15g}{16}$

$Mg - N = M\frac{15g}{16} \Rightarrow N = \frac{Mg}{16}$

Q.4 In a Young's double slit experiment, the slit separation d is 0.3 mm and the screen distance D is 1m. A parallel beam of light of wavelength 600 nm is incident on the slits at angle α as shown in figure. On the screen, the point O is equidistant from the slits and distance PO is 11.0 mm. which of the follownig statement(s) is/are correct ?



(1) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point O

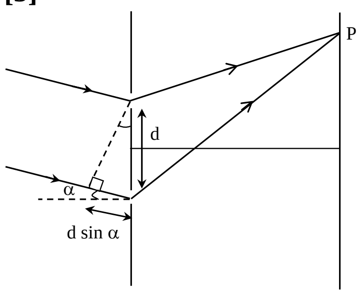
(2) For $\alpha = 0$, there will be constructive interference at point P

(3) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point P.

(4) Fringe spacing depends on α.

Ans. **[3]**

Sol.



Path difference at p = $d\sin\alpha + \frac{dy}{D}$ $\alpha \to$ small $\sin\alpha = \alpha$

$= d\alpha + \frac{dy}{D}$

$= (0.3 \times 10^{-3})\left[\alpha + \frac{11\times10^{-3}}{1}\right]$

Option (3) $\frac{3\times10^{-4}\left[\frac{0.36}{\pi}\frac{\pi}{180}+11\times10^{-3}\right]}{600\times10^{-9}} = \frac{10^3}{2}\left[13\times10^{-3}\right] = \frac{13}{2}$

$\frac{\Delta}{\lambda} = \frac{13}{2} = 6.5$ desttructive interferace.

(2) $\alpha = 0$, $\frac{\Delta}{\lambda} = \frac{3\times10^{-4}\times11\times10^{-3}}{6\times10^{-7}} = \frac{11}{2} = 5.5$(Destructive)

$\frac{\Delta}{\lambda} = \frac{d\alpha}{\lambda} = \frac{3\times10^{-4}\times2\times10^{-3}}{\lambda} = \frac{6\times10^{-7}}{6\times10^{-7}} = 1$(Constructive)

Q.5 A mixture of ideal gas containing 5 moles of monatomic gas and 1 mole of rigid diatomic gas is initially at pressure P_0, volume V_0, and temperature T_0. If the gas mixture is adiabatically compressed to a volume $V_0/4$, then the correct statement(s) is/are, (Given $2^{1.2} = 2.3$; $2^{3.2} = 9.2$: ; R is gas constant)

(1) The final presure of the gas mixture after compression is in betwen $9P_0$ and $10P_0$.

(2) The work |W| done during the proces is 13 RT_0.

(3) Adiabatic constant of the gas mixture is 1.6.

(4) The average kinetic energy of the gas mixture after compression is in betwen $18RT_0$ and $19RT_0$.

Ans. **[1,2,3]**

Sol. $\gamma_{mix} = \frac{n_1Cp_1 + n_2Cp_2}{n_1C_{V1} + n_2C_{V2}}$

$Cv_1 = \frac{3R}{2}$ $Cp_1 = \frac{5R}{2}$

$Cv_2 = \frac{5R}{2}$ $Cp_2 = \frac{7R}{2}$

$= \frac{5\left(\frac{5R}{2}\right) + (1)\left(\frac{7R}{2}\right)}{5\left(\frac{3R}{2}\right) + (1)\left(\frac{5R}{2}\right)} = \frac{32R/2}{20R/2} = \frac{8}{5}$

$P_0V_0^{\gamma} = P\left(\frac{V_0}{4}\right)^{\gamma}$ $P = P_0(4)^{\gamma}$ $P = P_0(4)^{8/5} = 9.2P_0$

W = $\frac{P_1V_1 - P_2V_2}{\gamma - 1}$

$= \frac{P_0V_0 - (4)^{8/5}P_0\frac{V_0}{4}}{\frac{8}{5} - 1}$

$= P_0V_0\left[\frac{1-4^{3/5}}{3/5}\right] = P_0V_0\frac{-1.29}{0.6} = -2.15P_0V_0$

$|W| = 2.15\ (6)\ RT_0 = 13RT_0$

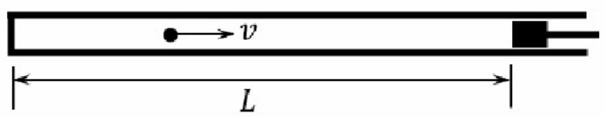
$9P_0 < P_f < 10P_0$ $|W| = 13\ RT_0$ $\gamma = \frac{8}{5} = 1.6$

$KE = \frac{f}{2}nRT$ $\gamma = 1 + \frac{2}{f} \Rightarrow \frac{f}{2} = \frac{1}{\gamma - 1}$

$= \frac{f}{2}PV = \frac{1}{3/5}(9.2P_0)\left(\frac{V_0}{4}\right) = \frac{5 \times 9.2 \times 6RT_0}{33 \times 4}$

$= 23RT_0$ options : (1) (2) (3)

Q.6 A small particle of mass m moving inside a heavy, hollow and stright tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low speed V and such that $V << \frac{dL}{L}v_0$, where dL is the infinitesimal displacement of the piston. Which of the following statement (s) is/are correct ?



(1) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2}L_0$.

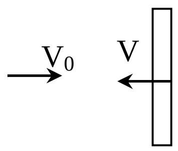
(2) After each collision with the piston, the particle speed increases by 2V.

(3) The rate at which the particle strikes the piston is v/L.

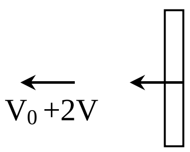
(4) If the piston moves inward by dL, the particle speed increases by $2v\frac{dL}{L}$.

Ans. **[1,2]**

Sol.



$e = 1 \Rightarrow V_{ap} = Vsep$



$V \rightarrow\ = $ small

Frequency of Collision $= \frac{1}{2L/V_0} = \frac{V_0}{2L}$

Speed of particle After 1st collision $= V_0 + 2\ V$

Speed of particle After 2nd collision $= V_0 + 4\ V$

Speed of particle After nth collision $= V_0 + 2\ nV$

time taken by piston to move $L_0/2 = \frac{L_0/2}{V}$

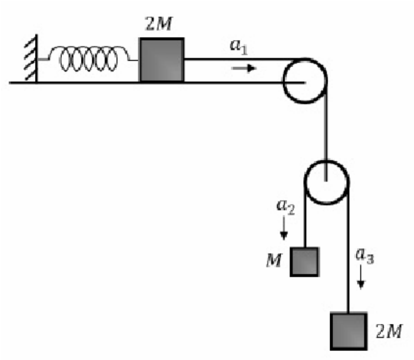
no of collision $= \frac{V_0}{2L} \times \frac{L_0}{2V} = \frac{L_0V_0}{4LV}$

For dL displacement $\Rightarrow t = \frac{dL}{V}$ $N = \frac{V_0}{2L_0}\frac{dL}{V}$

Speed = $V_0 + 2\left(\frac{V_0 dL}{2L_0 V}\right)V = V_0 + \frac{V_0 dL}{L}$

Option (1) and (2)

Q.7 Ablock of mass 2M is attached to a massless spring with spring-consant k. This block is connected to two other blocks of masses M and 2M using two massless pulleys and strings. The accelerations of the block sare a_1, a_2 and a_3 as shown in the figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is x_0. Which of the following ooptin (s) is/are corect ? [g is the accelration due to gravity. Neglect friction]



(1) When spring achieves an extension of $\frac{x_0}{2}$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\frac{M}{5k}}$.

(2) $x_0 = \frac{4Mg}{k}$

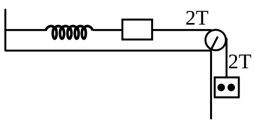
(3) At an extension of $\frac{x_0}{4}$ of the spring, the magnitude of accelration of the block connected to the spring is $\frac{3g}{10}$.

(4) $a_2 - a_1 = a_1 - a_3$

Ans. **[4]**

Sol. $a_p = \frac{a_2 + a_3}{2}$ or $a_2 - a_1 = a_1 - a_3$

Also

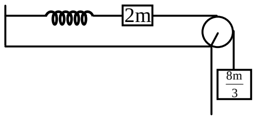


Reduced mass = $\frac{m \times 2m}{m + 2m} = \frac{2m}{3}$

Tension = $\frac{2m_1 m_2}{m_1 + m_2} g_{eff} = \frac{4m}{3} g_{eff}$

$2T = 2 \times \frac{4m}{3} g_{eff}$

$$\frac{2T}{g_{eff}} = \frac{8m}{3}$$



$$\frac{8m}{3} . x_0 g + = \frac{1}{2} k x_0^2$$

$$x_0 = \frac{16\,mg}{3k}$$

Energy conservation

$$\frac{8m}{3} g \frac{x_0}{2} = \frac{1}{2} k \frac{x_0^2}{4} + \frac{1}{2}(2m)v^2$$

$$\frac{16mg}{3k}\frac{4mg}{3} - \frac{1}{8} k \left(\frac{16mg}{3k}\right)^2 = mv^2 + \frac{4m}{3} v^2$$

$$\frac{32}{9}\frac{m^2 g^2}{k} = \frac{7}{3} mv^2$$

$$v^2 = \frac{32}{21}\frac{mg^2}{k}$$

$$v = g\sqrt{\frac{32}{21}\frac{mg}{k}}$$

Q.8 A free hydrogen atom after absorbing a photon of wavelength λ_a gets excited from the state n = 1 to the state n = 4. Immediately after that the electron jumps to n = m state by emitting a photon of wavelegnth λ_0. Let the change in momentum of atom due to the adsorption and the emission are Δp_a and Δp_e, respectively. If $\lambda_a/\lambda_e = \frac{1}{5}$, which of the option (s) is/are correct ? [Use hc = 1242 eV nm; 1nm = 10^{-9} m, h and c are Planck's consatnt and speed of light, resepctively]

(1) λ_e = 418 nm

(2) The ratio of kinetic energy fo the electron in the sate n = m to the state n = 1 is $\frac{1}{4}$.

(3) m = 2

(4) $\Delta p_a/\Delta p_e = \frac{1}{2}$

Ans. **[2,3]**

Sol. $\lambda \propto \frac{1}{\Delta E}$

$$\frac{\lambda_\alpha}{\lambda_e} = \frac{\frac{1}{m^2} - \left(\frac{1}{4}\right)^2}{1 - \left(\frac{1}{4}\right)^2} = \frac{1}{5}$$

$$\frac{1}{m^2} - \frac{1}{16} = \frac{1}{5} \times \frac{15}{16} = \frac{3}{16}$$

$$\frac{1}{m^2} = \frac{4}{16} \Rightarrow m = 2$$

$$\frac{hc}{\lambda_e} = 13.6\left(\frac{1}{4} - \frac{1}{16}\right)$$

$$\frac{1240}{\lambda_e} = 13.6\left(\frac{12}{16 \times 4}\right)$$

$$\lambda_e = \frac{1240 \times 16 \times 4}{12 \times 13.6} = 487\,\text{nm}$$

$$KE \propto \frac{1}{n^2} \Rightarrow \frac{KE_{m=2}}{KE_{m=1}} = \frac{1}{4}$$

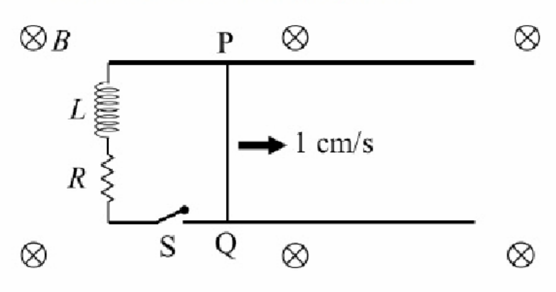
SECTION – 2 (Maximum Marks : 18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value ot TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme.

Full Marks : +3 If ONLY the correct numerical value is entered as answer.
Zero Marks : 0 In all other cases.

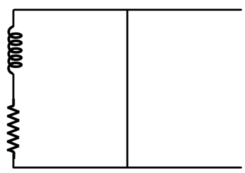
Q.1 A 10 cm long perfectly conducting wire PQ is moving with a velocity 1 cm/s on a pair of horizontal rails of zero ressistance. One side of the rails is connected to an inductor L = 1 mH and a resistance R = 1Ω as shown in figure. The horizontal rails, L and R lie in the same plane with a uniform magnetic field B = 1T perpendicular to the plan. If the key S is closed at certain instant, the current in the circuit after 1 millisecond is $x \times 10^{-3}$ A, where the value of x is ………. .

[Assume the velocity of wire PQ remains constant (1 cm/s) after key S is closed. Given : $e^{-1} = 0.37$, where e is base of the natural logarithm]



Ans. **[0.63]**

Sol.



$E = v\ell B = (10^{-2}\text{ m/s})\left(\frac{10}{100}\right)(1) = 10^{-3}$ volt

$i = i_0\left(1 - e^{-t/\tau}\right)$ $\tau = \frac{L}{R} = \frac{10^{-3}}{1}$

$= \frac{10^{-3}}{1}\left[1 - e^{-10^{-3}/10^{-3}}\right]$

$= 10^{-3}\left(1 - \frac{1}{e}\right) = 0.63 \times 10^{-3}$

Q.2 Suppose a $^{226}_{88}Ra$ nucleus a rest and in ground state undergoes α-decay to a $^{222}_{86}Rn$ nucleus in its excited state. The kinetic energy of the emitted a particle is found to be 4.44 MeV. $^{222}_{86}Rn$ nucleus then goes to its ground state by γ-decay. The energy of the emitted γ photon is ………….. keV.

[Given : atomic mass of $^{226}_{88}Ra$ = 226.005 u, atomic mass of $^{222}_{86}Rn$ = 222.000 u, atomic mass of α particle = 4.000 u, 1 u = 931 MeV/c^2, c is speed of the light]

Ans. **[135]**

Sol.

$^{226}_{88}Ra \to ^{222}_{86}Rn^* + ^{4}_{2}He$

↓ 4.44 MeV

$^{222}_{86}Rn + \gamma$ decay

Mass defect Δm = (226.005) – [222.000 + 4.000]

= 226.005 – 226.000 = 0.005

Q – value = 0.005 × 931.5 = 4.6575 MeV

$\overrightarrow{P_{Rn}} = -\overrightarrow{P_{He}} \Rightarrow KE \propto \frac{1}{m}$

$\frac{(KE)_{Rn}}{(KE)_\alpha} = \frac{m_\alpha}{m_{Rn}} = \frac{4.000}{222.000} \Rightarrow (KE)_{Rn} = \frac{4}{222} \times 4.44$ MeV

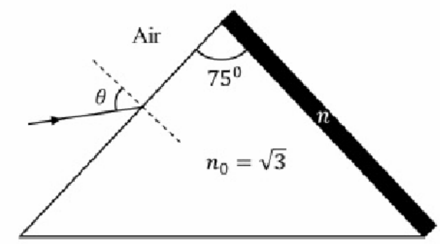
$(KE)_{Rn} = 0.0799 \simeq 0.08$ MeV

Energy of γ particale = 4.65 – [4.44 + 0.08]

= 0.135 MeV

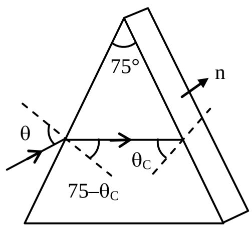
= 135 KeV

Q.3 A monochromatic light is incident from air on a refracting surface of a prism of angle 75º and refractive index $n_0 = \sqrt{3}$. The other refracting surface of the prism is coated by a thin film of material of refractive index n as shown in figure. The light suffers total internal reflection at the coated prism surface for an incidence angle of θ ≤ 60º. The value of n^2 is …………… .



Ans. **[1.5]**

Sol.



$\sin\theta_C = \frac{n}{\sqrt{3}}$

$\sin\theta = \sqrt{3}\sin(75 - \theta_c)$

$\theta = 60°, \frac{1}{2} = \sin(75 - \theta_c)$

$\sin 30° = \sin(75 - \theta_c)$

$75 - \theta_c = 30 \Rightarrow \theta_c = 45°$

$\frac{n}{\sqrt{3}} = \frac{1}{\sqrt{2}} \Rightarrow n = \frac{\sqrt{3}}{\sqrt{2}} \Rightarrow n^2 = \frac{3}{2}$

$n^2 = 1.50$

Q.4 An optical bench has 1.5 m long scale having four equal divisions in each cm. While measurnig the focal length of a convex lens, the lens is kept at 75 cm mark of the scale and the object pin is kept at 45 cm mark. The image of the object pin on the other side of the lens overlaps with image pin that is kept at 135 cm mark. In this experiment, the percentage error in the measurement of the focal length of the lens is ……………….. .

Ans. **[1.38 or 1.39]**

Sol. Least Count = $\frac{1\text{cm}}{4}$

Object distance $u = u_{lens} - u_{pin} \Rightarrow du = du_1 + du_2$

$= 75 - 45 = 30$ cm

$du = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$dv = \frac{1}{2}$

$v = (135 - 75)\text{cm} = 60$ cm

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{60} - \frac{1}{-30} = \frac{1}{f}$

$f = 20$ cm

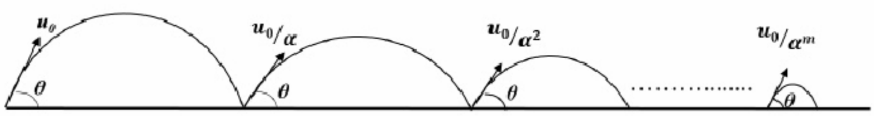
$\frac{dv}{v^2} + \frac{du}{u^2} = \frac{df}{f^2} \Rightarrow \frac{1/2}{(60)^2} + \frac{1/2}{(30)^2} = \frac{df}{(20)^2}$

$df = \frac{1}{2} \times 400 \left[\frac{6}{(60)^2} + \frac{1}{(30)^2}\right]$

$= \frac{5}{36} \times 2$

$\frac{df}{f} \times 100 = \frac{10/36}{20} \times 100 = \frac{50}{36} = 1.388$

Q.5 A ball is thrown from ground at an angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magntiude of average velocity of the ball up to the point when it hits the ground for the first time is V_1. After hitting the ground, the ball rebounds at the same angle θ but with a reduced speed of u_0/α. Its motion continues for a long time as shown in figure. If the magnhitude of average velocity of the ball for entire duriation of motion is 0.8 V_1, the value of α is …………… .



Ans. **[4]**

Sol. $V_{avg} = \dfrac{\text{Total displacement}}{\text{Time taken}}$

$$= \frac{2\left[\dfrac{u_0 \cos\theta\, u_0 \sin\theta}{g}\right] + 2\left[\dfrac{\dfrac{u_0}{\alpha}\cos\theta \dfrac{u_0}{\alpha}\sin\theta}{g}\right] + \ldots\ldots}{\dfrac{2u_0 \sin\theta}{g} + \dfrac{2\dfrac{u_0}{\alpha}\sin\theta}{g} + \ldots..}$$

$$= \frac{u_0 \cos\theta\left[1 + \dfrac{1}{\alpha^2} + \dfrac{1}{\alpha^4} + \ldots.. + \dfrac{1}{\alpha^{2m}}\right]}{1 + \dfrac{1}{\alpha} + \dfrac{1}{\alpha^2} + \ldots.. + \dfrac{1}{\alpha^m}}$$

$m \to \infty$

$$= V_1\left[\frac{1 + \dfrac{1}{\alpha^2} + \dfrac{1}{\alpha^4} + \ldots..}{1 + \dfrac{1}{\alpha} + \dfrac{1}{\alpha^2} + \ldots..}\right]$$

$$= V_1\left[\frac{\dfrac{1}{1 - \dfrac{1}{\alpha^2}}}{\dfrac{1}{1 - \dfrac{1}{\alpha}}}\right] = v_1\left[\frac{\dfrac{\alpha^2}{\alpha^2 - 1}}{\dfrac{\alpha}{\alpha - 1}}\right]$$

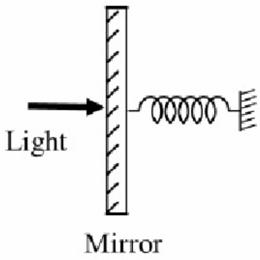
$$V_{avg} = V_1\left[\frac{\alpha^2}{\alpha^2 - 1}\,\frac{\alpha - 1}{\alpha}\right] = \frac{\alpha}{\alpha + 1}\; V_1$$

$$0.8V_1 = V_1 = \frac{\alpha}{\alpha + 1} \Rightarrow \alpha = 0.8\alpha + 0.8$$

$0.2\alpha = 0.8$

$\alpha = 4$

Q.6 A perfectly reflecting mirror of mass M mounted on a spring constitues a spring-mass system of angular frequency Ω such that $\dfrac{4\pi MW}{h} = 10^{24}\ m^{-2}$ with h as Planck's constant. N photons of wavelength $\lambda = 8\pi \times 10^{-6}$ m strike the mirror simultaneously at normal incidence such that the mirror gets displaced by 1μm. If the value of N is $x \times 10^{12}$, then the value of x is …………… . [Consider the spring as massless]



Ans. **[1]**

Sol. A = 1 μm N = No. of photons

$Ma\omega = Mv_{max} = 2N\frac{h}{\lambda}$

$M(10^{-6})(\Omega) = 2N\frac{h}{8\pi\times10^{-6}}$

$10^{-12} = \frac{Nh}{4\pi M\Omega}$

$N = \left(\frac{4\pi M\Omega}{h}\right)10^{-12}$

$N = 10^{24}\times10^{-12} = 10^{12}$

SECTION – 3 (Maximum Marks : 12)

- This section contains **Two (02)** List-Match sets.
- Each List-Match set has **TWO (02)** Multiple Choice Questions.
- Each List-Match set has two lists : **List-I** and **List-II**
- **List-I** has **Four** entries (I), (II), (III) and (IV) and List-II has Six entries (P), (Q), (R), (S), (T) and (U).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- For each question, marks will be awarded according to the following marking scheme :

  Full Marks : +3 If ONLY the option corresponding to the correct matching is chosen.

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered.)

  Negative Marks : –1 In all other cases.

Answer the following by appropriately matching the lists based on the inforamtion given in the paragraph.

Q.1 A musical instrument is made using four different metal strings, 1, 2, 3 and 4 with mass per unti length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while List-II lists the magnitude of some quantity.

	LIST-I		**LIST-II**
(I)	String-1 (μ)	**(P)**	1
(II)	String-2 (2μ)	**(Q)**	1/2
(III)	String-3 (3μ)	**(R)**	$1/\sqrt{2}$
(IV)	String-4 (4μ)	**(S)**	$1/\sqrt{3}$
		(T)	3/16
		(U)	1/16

If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be,

(1) I → Q; II → S, III → R ; IV → P

(2) I → P; II → R, III → S ; IV → Q

(3) I → P; II → Q, III → T ; IV → S

(4) I → Q; II → P, III → R ; IV → T

Ans. **[2]**

Sol. $f_0 = \frac{1}{2L_0}\sqrt{\frac{T}{\mu}}$

String (1) $f_0 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}} \rightarrow P$

String (2) $f_2 = \frac{1}{2L_0}\sqrt{\frac{T_0}{2\mu}} = \frac{f_0}{\sqrt{2}} \rightarrow R$

String (3) $f_3 = \frac{1}{2L_0}\sqrt{\frac{T}{3\mu}} = \frac{f_0}{\sqrt{3}} \rightarrow S$

String (4) $f_4 = \frac{1}{2L_0}\sqrt{\frac{T}{4\mu}} = \frac{f_0}{2} \rightarrow Q$

$I \rightarrow P, II \rightarrow R, III \rightarrow S, IV \rightarrow Q$

Q.2 **Answer the following by appropriately matching the lists based on the inforamtion given in the paragraph.**

A musical instrument is made using four different metal strings, 1, 2, 3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ resopectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frenecy is f_0.

List-I gives the above four stirngs while list-II lists the magnitude of some quantity.

	LIST-I		LIST-II
(I)	String-1 (μ)	**(P)**	1
(II)	String-2 (2μ)	**(Q)**	1/2
(III)	String-3 (3μ)	**(R)**	$1/\sqrt{2}$
(IV)	String-4 (4μ)	**(S)**	$1/\sqrt{3}$
		(T)	3/16
		(U)	1/16

The length of strings 1, 2, 3 and 4 are kept fixed at L_0, $\frac{3L_0}{2}$, $\frac{5L_0}{4}$ and $\frac{7L_0}{4}$, respectively. String 1, 2, 3 and 4 are vibrated at their 1^{st}, 3^{rd}, 5^{th}, and 14^{th} harmonics, respectively such that all the strings have same frequency. The correct match for the tension in the four strings in the untis of T_0 will be.

(1) $I \rightarrow P; II \rightarrow Q, III \rightarrow T ; IV \rightarrow U$

(2) $I \rightarrow P; II \rightarrow Q, III \rightarrow R ; IV \rightarrow T$

(3) $I \rightarrow T; II \rightarrow Q, III \rightarrow R ; IV \rightarrow U$

(4) $I \rightarrow P; II \rightarrow R, III \rightarrow T ; IV \rightarrow U$

Ans. **[1]**

Sol. $f_0 = \frac{1}{2L_0}\sqrt{\frac{T}{\mu}}$

$$f_0 = \frac{3}{2\left[\frac{3L_0}{2}\right]}\sqrt{\frac{T_2}{2\mu}} = \frac{1}{\sqrt{2}}\frac{1}{L_0}\sqrt{\frac{T_2}{\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

$$T_2 = \frac{T_0}{2}$$

$$f_0 = \frac{5}{2\left[\frac{5L_0}{4}\right]}\sqrt{\frac{T_3}{3\mu}} = \frac{2}{\sqrt{3}}\frac{1}{L_0}\sqrt{\frac{T_3}{\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

$$T_3 = \frac{3}{16}T_0$$

$$f_0 = \frac{14}{2\left[\frac{7L_0}{4}\right]}\sqrt{\frac{T_4}{4\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

$$\sqrt{4T_4} = \frac{\sqrt{T_0}}{2} \Rightarrow T_4 = \frac{T_0}{16}$$

I $\rightarrow$ P, II $\rightarrow$ Q, III $\rightarrow$ T, IV $\rightarrow$ U

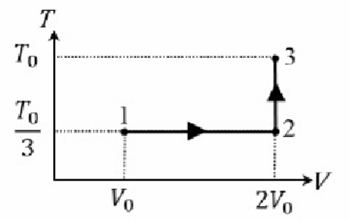
Q.3 Answer the following by appropriately matching the lists based on the inforamtion given in the paragraph.

In a thermodynamic process on an ideal monoatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the sytsem and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas $X = \frac{3}{2}R\ln\left(\frac{T}{T_A}\right) + R\ln\left(\frac{V}{V_A}\right)$. Here, R is gas constant, V is volume of gas, T_A and V_A ar constants.

The List-I below gives some quantities invovled in a process and List-II gives some possible values of these quantities.

	LIST-I		LIST-II
(I)	Work done by the system in process $1 \rightarrow 2 \rightarrow 3$	**(P)**	$\frac{1}{3}RT_0\ln2$
(II)	Change in internal energy in process $1 \rightarrow 2 \rightarrow 3$	**(Q)**	$\frac{1}{3}RT_0$
(III)	Heat absorbed by the system in process $1 \rightarrow 2 \rightarrow 3$	**(R)**	RT_0
(IV)	Heat absorbed by the system in process $1 \rightarrow 2$	**(S)**	$\frac{4}{3}RT_0$
		(T)	$\frac{1}{3}RT_0(3 + \ln 2)$
		(U)	$\frac{5}{6}RT_0$

If the process on one mole of monatomic ideal gas is as shown in the TV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is.



(1) I $\rightarrow$ P; II $\rightarrow$ R, III $\rightarrow$ T ; IV $\rightarrow$ P
(2) I $\rightarrow$ P; II $\rightarrow$ T, III $\rightarrow$ Q ; IV $\rightarrow$ T
(3) I $\rightarrow$ P; II $\rightarrow$ R, III $\rightarrow$ T ; IV $\rightarrow$ S
(4) I $\rightarrow$ S; II $\rightarrow$ T, III $\rightarrow$ Q ; IV $\rightarrow$ U

Ans. **[1]**

Sol. (1) $W_{123} = nR\,\frac{T_0}{3}\,\ell n2 + 0 = \frac{RT_0}{3}\,\ell n^2$ $1 \rightarrow P$

(2) $dU_{123} = dU_{12} + dU_{23}$

$= 0 + nCv\,dT$

$= 0 + 1 \times \frac{3R}{2}\left(T_0 - \frac{T_0}{3}\right) = RT_0$

$2 \rightarrow R$

(3) $dQ_{123} = dQ_{12} + dQ_{23}$

$= \frac{RT_0}{3}\,\ell n2 + nC_V\,dT$

$= \frac{RT_0}{3}\,\ell n2 + RT_0$

$= \frac{RT_0}{3}(\ell n2 + 3)$

$3 \rightarrow T$

(4) $dQ_{12} = dW$

$= \frac{RT_0}{3}\,\ell n2$

$4 \rightarrow P$

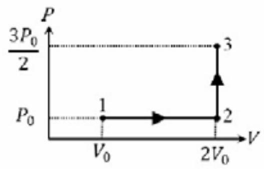
Q.4 **Answer the following by appropriately matching the lists based on the inforamtion given in the paragraph.**

In a thermodynamic process on an ideal monoatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the sytsem and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas $X = \frac{3}{2}R\ln\left(\frac{T}{T_A}\right) + R\ln\left(\frac{V}{V_A}\right)$. Here, R is gas constant, V is volume of gas, T_A and V_A ar constants.

The List-I below gives some quantities invovled in a oproces and List-II gives some possible values of these quantities.

	LIST-I		LIST-II
(I)	Work done by the system in process $1 \rightarrow 2 \rightarrow 3$	**(P)**	$\frac{1}{3}RT_0\ln 2$
(II)	Change in internal energy in process $1 \rightarrow 2 \rightarrow 3$	**(Q)**	$\frac{1}{3}RT_0$
(III)	Heat absorbed by the system in process $1 \rightarrow 2 \rightarrow 3$	**(R)**	RT_0
(IV)	Heat absorbed by the system in process $1 \rightarrow 2$	**(S)**	$\frac{4}{3}RT_0$
		(T)	$\frac{1}{3}RT_0(3 + \ln 2)$
		(U)	$\frac{5}{6}RT_0$

If the process carried out on one mole of monatomic ideal gas is as shown in the PV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is.



(1) I $\to$ Q; II $\to$ R, III $\to$ S ; IV $\to$ U

(2) I $\to$ Q; II $\to$ S, III $\to$ R ; IV $\to$ U

(3) I $\to$ S; II $\to$ R, III $\to$ Q ; IV $\to$ T

(4) I $\to$ Q; II $\to$ R, III $\to$ P ; IV $\to$ U

Ans. **[1]**

Sol. (1) $W_{123} = W_{12} + W_{23}$

$= PdV + 0$

$= P_0 V_0 = \frac{RT_0}{3}$

(2) $dU_{123} = dU_{12} + du_{23}$

$= nC_v (T_3 - T_1)$

$= n\frac{3}{2}R(T_3 - T_1)$

$= \frac{3}{2}(RT_3 - RT_1)$

$= \frac{3}{2}(3P_0V_0 - P_0V_0) = 3 P_0V_0 = RT_0$

$2 \to R$

(3) $dQ_{123} = dQ_{12} + dQ_{23}$

$= nCp (T_2 - T_1) + nC_V (T_3 - T_2)$

$= \frac{5}{2}(RT_2 - RT_1) + \frac{3}{2}(RT_3 - RT_2)$

$= \frac{5}{2}(2P_0V_0 - P_0V_0) + \frac{3}{2}(3P_0V_0 - 2P_0V_0)$

$= \frac{5}{2}P_0V_0 + \frac{3}{2}P_0V_0 = 4P_0V_0 = \frac{4}{3}P_0V_0$

$3 \to S$

(4) $dQ_{12} = 5P_0V_0 = \frac{5}{6}RT_0$

$4 \to U$