Memory Based Questions & Solutions

PAPER (पेपर)- 1 | SUBJECT : MATHEMATICS

PAPER-1 : INSTRUCTIONS TO CANDIDATES

- Question Paper-1 has three (03) parts: Physics, Chemistry and Mathematics.
- Each part has a total eighteen (18) questions divided into three (03) sections (Section-1, Section-2 and Section-3).
- Total number of questions in Question Paper-1 are Fifty Four (54) and Maximum Marks are One Hundred Eighty Six (186)

Type of Questions and Marking Schemes

SECTION-1 (Maximum Marks : 12)

- This section contains FOUR (04) questions.
- Each question has FOUR options. ONLY ONE of these four options is the correct answer.
- For each question, choose the correct option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  
<table>
<thead>
<tr>
<th>Full Marks</th>
<th>Partial Marks</th>
<th>Zero Marks</th>
<th>Negative Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>+3</td>
<td>+1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>(IF ONLY the correct option is chosen)</td>
<td>(IF the correct option is chosen)</td>
<td>(IF none of the options chosen)</td>
<td>(In all other cases)</td>
</tr>
</tbody>
</table>

SECTION-2 (Maximum Marks : 32)

- This section contains EIGHT (08) questions.
- Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  
<table>
<thead>
<tr>
<th>Full Marks</th>
<th>Partial Marks</th>
<th>Zero Marks</th>
<th>Negative Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>+4</td>
<td>+3</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>(IF only the correct option(s) is (are) chosen)</td>
<td>(IF all the four options are correct but ONLY three options are chosen)</td>
<td>(IF none of the options chosen)</td>
<td>(In all other cases)</td>
</tr>
</tbody>
</table>

SECTION-3 (Maximum Marks : 18)

- This section contains SIX (06) questions. The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places truncate/round-off the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  
<table>
<thead>
<tr>
<th>Full Marks</th>
<th>Zero Marks</th>
<th>Negative Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>+3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(IF ONLY the correct numerical value is entered)</td>
<td>(In all other cases)</td>
<td></td>
</tr>
</tbody>
</table>

Answering Questions:

- To select the option(s), use the mouse to click on the corresponding button(s) of the option(s).
- To deselect the chosen option for the questions of SECTION-1, click on the button of the chosen option again or click on the Clear button.
Response button to clear the chosen option.

- To deselect the chosen option(s) for the questions of SECTION-2, click on the button(s) of the chosen option(s) again or click on the Clear Response button to clear all the chosen options.

- To change the option(s) of a previously answered question of SECTION-1 and SECTION-2 first deselect as given above and then select the new option(s).

- To answer questions of SECTION-3, use the mouse to click on numbers (and/or symbols) on the on-screen virtual numeric keypad to enter the numerical value in the space provided for answer.

- To change the answer of a question of SECTION-3, first click on the Clear Response button to clear the entered answer and then enter the new numerical value.

- To mark a question ONLY for review (i.e. without answering it), click on the Mark for Review & Next button.

- To mark a question for review (after answering it), click on Mark for Review & Next button – the answered question which is also marked for review will be evaluated.

- To save the answer, click on the Save & Next button – the answered question will be evaluated.
PART-III: MATHEMATICS

SECTION-1 (Maximum Marks : 12)

- This section contains FOUR (04) questions.
- Each question has FOUR options ONLY ONE of these four options is the correct answer.
- For each question, choose the correct option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  
  Full Marks : +3 if ONLY the correct option is chosen.
  Zero Marks : 0 if none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : -1 in all other cases.

1. A line $y = mx + 1$ meets the circle $(x - 3)^2 + (y + 2)^2 = 25$ at points $P$ and $Q$. If mid point of $PQ$ has abscissa of $-\frac{3}{5}$, then value of $m$ satisfies

   एक रेखा $y = mx + 1$ गुण ($(x - 3)^2 + (y + 2)^2 = 25$) को $P$ तथा $Q$ बिंदुओं पर काटती है। यदि $PQ$ का मध्य सिर्फ $-\frac{3}{5}$ का भूमि होता है तब $m$ सम्पूर्ण होता है।

   (A) $6 \leq m < 8$  
   (B) $2 \leq m < 4$  
   (C) $-3 \leq m < -1$  
   (D) $4 \leq m < 6$

   Ans.  
   (B)

   Sol.

   For point $R$, $x = \frac{-3}{5}$  
   $y = 1 - \frac{3m}{5}$

   $R\left(\frac{-3}{5}, 1 - \frac{3m}{5}\right)$

   slope of $CR = \frac{\frac{-3m}{5} + 2}{\frac{-3}{5} - \frac{1}{m}}$  
   $= \frac{15 - 3m}{-3 - 15 + \frac{m}{m}}

   15m - 3m^2 = 18

   $m^2 - 5m + 6 = 0$

   $m = 2, 3$

   $2 \leq m \leq 4$
2. If \( z \) is a complex number belonging to the set \( S = \{ z : |z - 2 + i| \geq \sqrt{5} \} \) and \( z_0 \in S \) such that \( \frac{1}{|z_0 - 1|} \) is maximum. Then \( \arg \left( \frac{4 - z_0 - \bar{z}_0}{z_0 - \bar{z}_0 + 2i} \right) \) is

\[ \arg \left( \frac{4 - z_0 - \bar{z}_0}{z_0 - \bar{z}_0 + 2i} \right) \]

\( \text{Ans.} \: \frac{\pi}{4} \)

\( \text{Sol.} \: C \)

3. For \( z_0 - 1 \) to be minimum, \( z_0 = x_0 + iy_0 \) is at point P as shown in figure

\( \arg \left( \frac{4 - (z_0 - \bar{z}_0)}{2y + 2i} \right) = \arg \left( \frac{4 - 2x}{y + 2} \right) = -\frac{\pi}{2} \)

\( \text{Ans.} \: A \)

\( \text{Sol.} \:

\[ \begin{align*}
|z - (2 - i)| & \geq \sqrt{5} \\
\text{For } z_0 - 1 	ext{ to be minimum, } z_0 = x_0 + iy_0 \text{ is at point P as shown in figure} \\
\arg \left( \frac{4 - (z_0 - \bar{z}_0)}{2y + 2i} \right) & = \arg \left( \frac{4 - 2x}{y + 2} \right) = \frac{\pi}{2} \quad (\therefore x > 0)
\end{align*} \)

Area bounded the points \((x, y)\) in cartesian plane satisfying \(xy \leq 8\) and \(1 \leq y \leq x^2\) will be

\( \text{Ans.} \: \frac{16}{n^2} - \frac{14}{3} \)

\( \text{Sol.} \:

\begin{align*}
xy & \leq 8 \\
1 & \leq y \leq x^2 \\
x^2 \cdot x & = 8 \\
x & = 2
\end{align*} \)
4. \[ M = \begin{bmatrix} \sin^4 \theta & 1 - \sin^2 \theta \\ 1 + \cos^2 \theta & \cos^4 \theta \end{bmatrix} - \alpha I + \beta M^{-1} \]

Where \( \alpha = \alpha (\theta) \) and \( \beta = \beta (\theta) \) are real numbers and I is an identity matrix of 2x2.
If \( \alpha^* = \text{Min of set } \{ \alpha(\theta) : \theta \in [0,2\pi] \} \)
And \( \beta^* = \text{Min of set } \{ \beta(\theta) : \theta \in [0,2\pi] \} \)
Then value of \( \alpha^* + \beta^* \) is
\[ M = \begin{bmatrix} \sin^4 \theta & 1 - \sin^2 \theta \\ 1 + \cos^2 \theta & \cos^4 \theta \end{bmatrix} - \alpha I + \beta M^{-1} \]

\( \alpha^* = \alpha (\theta) \) and \( \beta^* = \beta (\theta) \) वास्तविक संख्याएं हैं। एक 2x2 का तत्समक आयतन है।
विधि \( \alpha^* = \text{मुख्यांक } \{ \alpha(\theta) : \theta \in [0,2\pi] \} \) का न्यूनतम
एवं \( \beta^* = \text{सह-मुख्यांक } \{ \beta(\theta) : \theta \in [0,2\pi] \} \) का न्यूनतम
tबने \( \alpha^* + \beta^* \) का मान लेंगे।

(A) \(-\frac{37}{16}\)  (B) \(-\frac{17}{16}\)  (C) \(-\frac{31}{16}\)  (D) \(-\frac{29}{16}\)

Ans.  (D)
Sol. \[ m = \sin^4 \theta + \cos^4 \theta + (1 + \sin^2 \theta)(1 + \cos^2 \theta) \]
\[ 2 + \sin^4 \theta + \cos^4 \theta = \begin{bmatrix} \sin^4 \theta & 1 - \sin^2 \theta \\ 1 + \cos^2 \theta & \cos^4 \theta \end{bmatrix} - \alpha I + \beta M^{-1} \]
\[ \begin{bmatrix} \sin^4 \theta & 1 - \sin^2 \theta \\ 1 + \cos^2 \theta & \cos^4 \theta \end{bmatrix} - \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} + \beta \begin{bmatrix} \cos^4 \theta & 1 - \sin^2 \theta \\ -\cos^4 \theta & \sin^4 \theta \end{bmatrix} \]
\[ \sin^4 \theta = \frac{\alpha + \beta}{\alpha} \cos^4 \theta, -1 - \sin^2 \theta = \frac{\beta}{\alpha} (1 + \sin^2 \theta) \]
\[ \beta = -\alpha \]
\[ \beta = -[\sin^4 \cos^4 + \sin^4 \cos^4 + 2] = -[\alpha + 2] \Rightarrow \beta = -\frac{37}{16} \]
5. \[ a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta} \] where \( \alpha \) and \( \beta \) are roots of equation \( x^2 - x - 1 = 0 \) and \( b_n = a_{n+1} - a_n \). Then

\[ (A) \ b_n = \alpha^{n+1} + \beta^n \quad (B) \ \sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{99} \quad (C) \ \sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{99} \quad (D) \ a_1 + a_2 + \ldots + a_n = a_{n+1} - 1 \]

Answ. (ACD)

Sol. (A) \[ b_n = \alpha^{n+1} + \beta^n \]

\[ \frac{a_n}{\alpha - \beta} = \frac{\alpha^{n+1} - \beta^{n+1}}{\alpha - \beta} = \frac{(\alpha^2 + 1) - \beta^{n+1}(\beta^2 + 1)}{\alpha - \beta} \]

\[ \frac{\alpha^{n+1}}{\alpha - \beta} = \frac{5 + \sqrt{5}}{2} \]

\[ \frac{\beta^n}{\alpha - \beta} = \frac{5 - \sqrt{5}}{2} \]

\[ a_n = \frac{\sqrt{5} \alpha^n + \sqrt{5} \beta^n}{\alpha - \beta} = \alpha^n + \beta^n \]

\[ \sum_{n=1}^{\infty} \frac{b_n}{10^n} = \sum_{n=1}^{\infty} \frac{\alpha^n}{10} - \sum_{n=1}^{\infty} \frac{\beta^n}{10} = \frac{10}{1 - \alpha/10} - \frac{10}{1 - \beta/10} = \frac{\alpha}{10 - \alpha} + \frac{\beta}{10 - \beta} \]

\[ \frac{10(\alpha + \beta) - 20}{100 - 10(\alpha + \beta) + 10\alpha} = \frac{10}{100} = \frac{12}{99} \]

\[ \sum_{n=1}^{\infty} \frac{a_n}{10^n} = \sum_{n=1}^{\infty} \frac{a_n}{(\alpha - \beta)10^n} = \frac{1}{\alpha - \beta} \left( \frac{\alpha}{10} - \frac{\alpha}{10} \right) = \frac{1}{\alpha - \beta} \left( \frac{1}{10} - \frac{1}{10} \right) = \frac{1}{\alpha - \beta} \left( \frac{1}{10} - \frac{1}{10} \right) = \frac{10}{99} \]

Option (C) is correct.

6. If a matrix \( M \) is given by \[
\begin{bmatrix}
0 & 1 & 2 \\
1 & 2 & 3
\end{bmatrix}
\]
and if \( M = \begin{bmatrix} 2 & 1 \\ 3 & 3 \end{bmatrix} \), then

\[ \text{adj}(M^3) + (\text{adj}M)^3 = -M \]

\[ a + 2b + \gamma = 2 \]

\[ a + 2b + \gamma = 3 \]

\[ a = 1, \quad b = -2, \quad \gamma = 1 \]

\[ |M| = -2 \]

\[ |\text{adj}M| = |M|^2 = |M|^4 = 16 \]

\[ |\text{adj}(M^3)| = |M^3| = -M \]

Answ. (AC)

Sol. \[
\begin{bmatrix}
0 & 1 & 2 \\
1 & 3 & 1 \\
3 & 1 & 1
\end{bmatrix}
\]

\[ \Rightarrow b + 2 \gamma = 1 \]

\[ a + 2b + \gamma = 2 \]

\[ 3a + b + \gamma = 3 \]

\[ a = 1, \quad b = -1, \quad \gamma = 1 \]

\[ |M| = -2 \]

\[ |\text{adj}M| = |M|^2 = |M|^4 = 16 \]

\[ |\text{adj}(M^3)| = |M^3| = -M \]
7. There are three bags $B_1$, $B_2$, $B_3$. $B_1$ contains 5 red and 5 green balls. $B_2$ contains 3 red and 5 green balls and $B_3$ contains 5 red and 3 green balls. Bags $B_1$, $B_2$, and $B_3$ have probabilities $\frac{3}{10}$, $\frac{3}{10}$, and $\frac{4}{10}$ respectively of being chosen. A bag is selected at random and a ball is randomly chosen from the bag. Then which of the following options is/are correct?

(A) Probability that the chosen ball is green equals $\frac{39}{80}$

(B) Probability that the chosen ball is green, given that the selected bag is $B_1$, equals $\frac{3}{8}$

(C) Probability that the selected bag is $B_3$, given that the chosen ball is green, equals $\frac{4}{13}$

(D) Probability that the selected bag is $B_3$, given that the chosen ball is green, equals $\frac{3}{10}$

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**Toll Free**: 1800 258 5555 | **Pan No.**: 09417100133 | 09417100134

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**JEE (ADVANCED) 2019 | DATE : 27-05-2019 | PAPER-1 | MEMORY BASED | MATHEMATICS**

There are three bags: $B_1$, $B_2$, $B_3$. $B_1$ has 5 red and 5 green balls. $B_2$ has 3 red and 5 green balls, and $B_3$ has 5 red and 3 green balls. Each bag has a probability of being chosen: $\frac{3}{10}$, $\frac{3}{10}$, and $\frac{4}{10}$.

(A) The probability of selecting a red ball from $B_1$ is $\frac{39}{80}$.

(B) The probability of selecting a red ball from $B_3$ is $\frac{3}{8}$.

(C) The probability of selecting a green ball from $B_2$ is $\frac{4}{13}$.

(D) The probability of selecting a bag with at least one green ball is $\frac{3}{10}$.

**Ans. Sol.**

<table>
<thead>
<tr>
<th>Bag 1</th>
<th>Bag 2</th>
<th>Bag 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red Balls</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>Green Balls</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Total</td>
<td>10</td>
<td>8</td>
</tr>
</tbody>
</table>

(A) \( P(\text{Ball is Green}) = P(B_1)P(G/B_1) + P(B_2)P(G/B_2) + P(B_3)P(G/B_3) \)

\[ = \frac{3}{10} \cdot \frac{5}{10} + \frac{3}{10} \cdot \frac{5}{10} + \frac{4}{10} \cdot \frac{3}{10} = \frac{39}{80} \]

(B) \( P(\text{Ball chosen is Green} | \text{Ball is from 3rd Bag}) = \frac{3}{8} \)

(C) \( P(\text{Ball is from 3rd Bag | Ball chosen is Green}) \)

\[ = \frac{P(B_3)P(G/B_3)}{P(B_1)P(G/B_1) + P(B_2)P(G/B_2) + P(B_3)P(G/B_3)} \]

\[ = \frac{\frac{3}{10}}{\frac{39}{80}} = \frac{4}{10} \]

\[ = \frac{4}{10} \cdot \frac{3}{10} + \frac{3}{10} \cdot \frac{5}{10} + \frac{4}{10} \cdot \frac{3}{10} = \frac{4}{13} \]
8. Let \( L_1 \) and \( L_2 \) denote the lines \( \vec{r} = \vec{a} + \lambda(\vec{b} \times \vec{c}) \), \( \lambda \in \mathbb{R} \) and \( \vec{r} = \mu(\vec{d} \times \vec{f}) \), \( \mu \in \mathbb{R} \) respectively. If \( L_3 \) is a line which is perpendicular to both \( L_1 \) and \( L_2 \) and cuts both of them, then which of the following options describe \( L_3 \)?

- \( \vec{r} = \vec{a} + \lambda(\vec{b} \times \vec{c}) \), \( \lambda \in \mathbb{R} \)
- \( \vec{r} = \frac{2}{9}(\vec{a} + \vec{b} + \vec{c}) + t(\vec{d} \times \vec{f}) \), \( t \in \mathbb{R} \)
- \( \vec{r} = \frac{1}{3}(\vec{a} + \vec{b} + \vec{c}) + t(\vec{d} \times \vec{f}) \), \( t \in \mathbb{R} \)

Ans. (B,C,D)

Sol. Both given lines are skew lines.

So direction ratios of any line perpendicular to these lines are \( 6i + 6j - 3k \) or \( 2i, 2j, -1k \).

Points at shortest distance between given lines are

\[ A(1-i, 2i, 2i) \]
\[ B(2i, -4j, 2i) \]

\[ \overrightarrow{AB} \perp L_1 \]
\[ \overrightarrow{AB} \perp L_2 \]

So \( \alpha = \frac{2}{9} \)

Now equation of required line \( \vec{r} = \left( \frac{8i}{9} + \frac{2j}{9} - \frac{k}{9} \right) + \alpha(2i + 2j - k) \)

Now by option B, C, D are correct.
9. Equation of ellipse \( E_1 \) is \( \frac{x^2}{9} + \frac{y^2}{4} = 1 \). A rectangle \( R_m \), whose sides are parallel to co-ordinate axes is inscribed in \( E_1 \) such that its area is maximum. Now \( E_n \) is an ellipse inside \( R_{n-1} \) such that its axes are along co-ordinate axes and has maximum possible area \( \forall m \geq 2, n \in \mathbb{N} \), further \( R_n \) is a rectangle whose sides are parallel to co-ordinate axes and is inscribed in \( E_{n-1} \), having maximum area \( \forall n \geq 2, n \in \mathbb{N} \).

(A) \( \sum_{m=1}^{n} \) area of rectangle \( (R_m) < 24 \ \forall m \in \mathbb{N} 

(B) Length of latus rectum of \( E_n \) is \( \frac{1}{6} \)

(C) Distance between focus and centre of \( E_m \) is \( \frac{\sqrt{5}}{32} \)

(D) The eccentricities of \( E_{18} \) and \( E_{19} \) are not equal

Ans. (AB)

Sol.

Area Max when \( \theta = 45^\circ 

\[
\begin{array}{c|c|c|c|c|c}
 & a & b & c & d & e \\
\hline
E_1 & 3 & 2 & 0 & 0 & 0 \\
E_2 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 & 0 \\
E_3 & \frac{\sqrt{2}}{3} & \frac{\sqrt{2}}{3} & \frac{\sqrt{2}}{3} & 0 & 0 \\
E_4 & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 \\
E_5 & \frac{\sqrt{2}}{5} & \frac{\sqrt{2}}{5} & \frac{\sqrt{2}}{5} & \frac{\sqrt{2}}{5} & \frac{\sqrt{2}}{5} \\
\end{array}
\]

(A) \( E_1 + E_2 + \ldots + E_n \)

\[
\frac{2ab}{1 - \frac{1}{\sqrt{2}}} = 4ab = 4.32 = 24
\]

(B) Length of LR is ellipse \( \frac{2b^2}{a} = \frac{2 \times 4.32^4}{2^4} = \frac{1}{6} \)

(C) distance between focus and center of ellipse \( a \), \( e_1 = \frac{3}{3} \times \frac{\sqrt{5}}{16} = \frac{\sqrt{5}}{16} \)

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10. In a non right angled triangle \( \triangle PQR \), let \( p, q, r \) denote the lengths of the sides opposite to the angle \( P, Q, R \) respectively. The median from \( R \) meets the side \( PQ \) at \( S \), the perpendicular from \( P \) meets the side \( QR \) at \( E \), and \( RS \) and \( PE \) intersect at \( O \). If \( p = \sqrt{3} \), \( q = 1 \) and the radius of the circumcircle of the \( \triangle PQR \) equals to 1, then which of the following options is/are correct?

(A) length of \( RS = \frac{\sqrt{3}}{2} \)

(B) length of \( OE = \frac{1}{6} \)

(C) Radius of incircle of \( \triangle PQR = \frac{\sqrt{3}}{2} (2 - \sqrt{3}) \)

(D) Area of \( \triangle SOE = \frac{\sqrt{3}}{12} \)

\( \text{A} \) एक अन्तर्वैदीक \( \triangle PQR \) में \( p, q, r \) क्रमांक कोणों \( P, Q, R \) के विकर्ण त्रिभुजों की लघुत्तम है। \( R \) से \( P \) की दिशा में भुजा लें \( PQ \) के \( S \) पर, \( P \) से \( QR \) को \( E \) पर मिलता है तथा \( RS \) एवं \( PE \) किन्हीं \( O \) पर प्रतिविद्यमान करती है। \( \text{विर} \) \( p = \sqrt{3}, q = 1 \) तथा \( \triangle PQR \) के परिवृत्त की विभाजन 1 हो तो निम्न घाती भिन्नों में काम सकते हैं -
11. Let \( T \) denote a curve \( y = f(x) \) which is in the first quadrant and let the point \((1, 0)\) lie on it. Let the tangent to \( T \) at a point \( P \) intersect the \( y \)-axis at \( Y_T \) and \( PY_T \) has length 1 for each point \( P \) on \( T \). Then which of the following option may be correct?

- **A** \( y = \sqrt{1-x^2} \) \( \sqrt{1-x^2} \)
- **B** \( xy' - \sqrt{1-x^2} = 0 \)
- **C** \( y = -\sqrt{1-x^2} + \sqrt{1-x^2} \)
- **D** \( xy' + \sqrt{1-x^2} = 0 \)

**Ans.** (ABCD)

**Sol.**

Let \( f(x) \) be differentiation of \( f(x) \) equation of tangent

\[
y - f(a) = f'(a)(x-a)
\]

Put \( x = 0 \)

\[
y - f(a) = -af'(a)
\]

\[
y = f(a) - af'(a)
\]

\[
y_T = 0, (0, f(0) - a\cdot 0)
\]

\[
PY_T = \sqrt{a^2 + (f'(a))^2} = 1
\]

\[
a^2 + a^2(\frac{g'(a)}{g(a)})^2 = 1
\]

\[
(\frac{g'(a)}{g(a)})^2 = \frac{1-a^2}{a^2} = \frac{1-a^2}{a^2}
\]
\[
\int kf(x) \, dx = k \int f(x) \, dx
\]

Put \( \sqrt{1-x^2} = t \)

\[ y = \pm \int \frac{-x^2 \, dt}{1-t^2} = \pm \left( 1 - \frac{1}{2} \left( \frac{1-t}{1+t} + \frac{1+t}{1-t} \right) \right) + c = \pm \left( 1 - \frac{1}{2} \frac{(1+t)^2}{1-t^2} \right) + c = \pm \left( \sqrt{1-x^2} - \ln 1+\sqrt{1-x^2} \right) + c \]

\[ \text{Put } x = 1 \text{ and } y = 0 \implies c = 0 \]

**Problem 12.** Let \( f : \mathbb{R} \rightarrow \mathbb{R} \) be given by

\[
f(x) = \begin{cases} 
  x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0 \\
  x^2 - x + 1, & 0 \leq x < 1 \\
  (2/3)x^3 - 4x^2 + 7x - (8/3), & 1 \leq x < 3 \\
  (x-2)/(x-3) - x + (10/3), & x \geq 3
\end{cases}
\]

Then which of the following options is/are correct?

(A) \( f \) is onto
(B) \( f' \) is not differentiable at \( x = 1 \)
(C) \( f' \) has a local maximum at \( x = 1 \)
(D) \( f \) is increasing on \((0, 1)\)

**Solution.**

\[
f(x) = \begin{cases} 
  x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0 \\
  x^2 - x + 1, & 0 \leq x < 1 \\
  (2/3)x^3 - 4x^2 + 7x - (8/3), & 1 \leq x < 3 \\
  (x-2)/(x-3) - x + (10/3), & x \geq 3
\end{cases}
\]

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SECTION-3 (Maximum Marks : 18)

- This section contains SIX (06) questions. The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places truncate/round-off the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4  If ONLY the correct numerical value is entered.
  Zero Marks : 0  In all other cases.

13. \( I = \frac{1}{\pi} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{1 + \sin^2 x} \)

Then find \( 27I^2 \).

\( I = \frac{1}{\pi} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1 + \sin^2 x} dx \)

Ans. \((4)\)

Sol.

\( I = \frac{1}{\pi} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1 + \sin^2 x} dx \) \quad \text{(1)}

by \( a + b - x \) property

\( \frac{1}{\pi} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1 + \sin^2 x} dx = \frac{2}{\pi} \int_{0}^{\frac{\pi}{4}} \frac{1}{1 + \sin^2 x} dx \) \quad \text{(2)}

Adding (1) and (2)

\( 2I = \frac{2}{\pi} \int_{0}^{\frac{\pi}{4}} \frac{1}{1 + \sin^2 x} dx \quad \Rightarrow \quad I = \frac{1}{\pi} \int_{0}^{\frac{\pi}{4}} \frac{1}{1 + \sin^2 x} dx = \frac{1}{\pi} \int_{0}^{\frac{\pi}{4}} \sec^2 x dx \quad \Rightarrow \quad 1 - \int_{0}^{\frac{\pi}{4}} \frac{1}{2 - (2\cos^2 x - 1)} dx = \frac{1}{\pi} \int_{0}^{\frac{\pi}{4}} \sec^2 x dx = 2 - \frac{2}{3\sqrt{3}} x \quad \Rightarrow \quad I = \frac{1}{\pi} \int_{0}^{\frac{\pi}{4}} \sec^2 x dx = \frac{1}{2} - \frac{1}{3\sqrt{3}} x \quad \Rightarrow \quad I = \frac{1}{2} - \frac{1}{3\sqrt{3}} \)

Put \( \tan x = \frac{1}{\sqrt{3}} \sec^2 x dx = dt \)

\( = \frac{2}{\pi} \left[ \frac{1}{3\sqrt{3}} - 1 \right] \left[ \tan^{-1} \left( \frac{1}{\sqrt{3}} \right) \right] = \frac{2}{\pi} \left[ \frac{1}{3\sqrt{3} \pi} - - \frac{1}{\pi} \left( \frac{1}{\sqrt{3}} \right) \right] = \frac{2}{\pi} \left[ \frac{1}{3\sqrt{3}} \right] = \frac{2}{3\sqrt{3}} \)

Now \( 27I^2 = 27 \times \frac{4}{27} = 4 \)
14. Let the point B be the reflection of the point A(2, 3) with respect to the line 8x - 6y = -23 = 0. Let T and T9 be circles of radii 2 and 1 with centres A and B respectively. Let T be a common tangent to the circles T and T9 such that both the circles are on the same side of T. If C is the point of intersection of T and the line passing through A and B, then the length of the line segment AC is

\[ 8x - 6y - 23 = 0 \]

\[ \text{Ans.} \quad 10 \]

Sol.

\[ AL = \frac{16 - 18 - 23}{10} = \frac{5}{2} \]

\[ CB = 1 \]

\[ CA = 2 \]

\[ CA - 5 = 1 \]

\[ CA = 10 \]

15. If \( a, d \) denotes an A.P. with first term \( a \) and common difference \( d \). If the A.P. formed by intersection of three A.P.'s given by \( (1, 3), (2, 5) \) and \( (3, 7) \) is a new A.P. \( (A, D) \). Then the value of \( A + D \) is \( \text{Ans.} \quad 157 \)

\[ \text{Sol.} \]

First series is \( [1, 4, 7, 10, 13, \ldots \ldots.] \)

Second series is \( [2, 7, 12, 17, \ldots \ldots.] \)

Third series is \( [3, 10, 17, 24, \ldots \ldots.] \)

See the least number in the third series which leaves remainder 1 on dividing by 3 and leaves remainder 2 on dividing by 5.

\[ \Rightarrow 52 \text{ is the least number in the third series which leaves remainder 1 on dividing by 3 and leaves remainder 2 on dividing by 5} \]

\[ \text{Now,} A = 52 \]

\[ D \text{ is L.C.M. of} (3, 5, 7) = 105 \]

\[ \Rightarrow A + D = 52 + 105 = 157 \]
17. Equation of three lines \( r = \lambda \mathbf{i} + \mu \mathbf{j} + \gamma \mathbf{k} \) and a plane \( x + y + z = 1 \) are given then area of triangle formed by point of intersection of line and plane is \( \Delta \), then \((6\Delta)^2\) equals

\( r = \lambda \mathbf{i}, r = \mu \mathbf{j} + \mathbf{j}, r = \gamma \mathbf{k} + \mathbf{k} \) दीने रेखाओं के समीकरण हैं एवं \( x + y + z = 1 \) एक समतल का समीकरण है तो \( \Delta \) के लिए \((6\Delta)^2\) का मान क्या होगा?

Ans. \((0.75)\)

Sol.
Put \((\lambda, 0, 0)\) in \( x + y + z = 1 \) \(\Rightarrow \lambda = 1 \Rightarrow P(1, 0, 0)\)

Put \((\mu, \mu, 0)\) \(\Rightarrow 2\mu = 1 \Rightarrow \mathbf{Q}(\frac{1}{2}, \frac{1}{2}, 0)\)

Put \((\gamma, \gamma, \gamma)\) \(\Rightarrow \gamma = \frac{1}{3} \Rightarrow R(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})\)

Area of triangle \(PQR = \frac{1}{2} |PQ \times PR| = \frac{1}{2} \left| \begin{vmatrix} 1 & \frac{1}{2} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} \end{vmatrix} \right| = \frac{1}{12} |1 + 0 + 0| = \frac{1}{12} \Rightarrow (6\Delta)^2 = 0.75\)

18. That \(\omega \neq 1\) be a cube root of unity. Then minimum value of set \(\{(a + b\omega + c\omega^2)^2; a, b, c\text{ are distinct non zero integers}\}\) equals _________.

माना \(\omega \neq 1\) एक कवर समूह है। तो सूची \(\{(a + b\omega + c\omega^2)^2; a, b, c\text{ निर्दि: सभी एक समान नहीं हैं}\}\) का मूल्यांकन मान होगा जहां \(a, b, c\) किन्तु—मिन्नत अवश्य समिक्षित हैं।

Ans. \((3)\)

Sol.
\[(a + b\omega + c\omega^2)^2 = a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2]\]

it will be minimum when \(a = 1, b = 2, c = 3\)
so minimum value is 3.

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