

# FINAL JEE(Advanced) EXAMINATION - 2019

(Held On Monday 27<sup>th</sup> MAY, 2019)

PAPER-2

TEST PAPER WITH ANSWER

## PART-3 : MATHEMATICS

### SECTION-1 : (Maximum Marks: 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all ) the correct answer(s)
- Answer to each question will be evaluated according to the following marking scheme:
  - Full Marks* : +4 If only (all) the correct option(s) is (are) chosen.
  - Partial Marks* : +3 If all the four options are correct but **ONLY** three options are chosen.
  - Partial Marks* : +2 If three or more options are correct but **ONLY** two options are chosen and both of which are correct.
  - Partial Marks* : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option.
  - Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered).
  - Negative Marks* : -1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then
  - choosing **ONLY** (A), (B) and (D) will get +4 marks;
  - choosing **ONLY** (A) and (B) will get +2 marks;
  - choosing **ONLY** (A) and (D) will get +2 marks;
  - choosing **ONLY** (B) and (D) will get +2 marks;
  - choosing **ONLY** (A) will get +1 marks;
  - choosing **ONLY** (B) will get +1 marks;
  - choosing **ONLY** (D) will get +1 marks;
  - choosing no option (i.e. the question is unanswered) will get 0 marks, and
  - choosing any other combination of options will get -1 mark.

1. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be given by  $f(x) = (x - 1)(x - 2)(x - 5)$ . Define  $F(x) = \int_0^x f(t) dt$ ,  $x > 0$ . Then which of the following options is/are correct ?
- (1) F has a local minimum at  $x = 1$
  - (2) F has a local maximum at  $x = 2$
  - (3)  $F(x) \neq 0$  for all  $x \in (0, 5)$
  - (4) F has two local maxima and one local minimum in  $(0, \infty)$

**Ans. (1,2,3)**

2. For  $a \in \mathbb{R}$ ,  $|a| > 1$ , let  $\lim_{n \rightarrow \infty} \left( \frac{1 + \sqrt[3]{2} + \dots + \sqrt[3]{n}}{n^{7/3} \left( \frac{1}{(an+1)^2} + \frac{1}{(an+2)^2} + \dots + \frac{1}{(an+n)^2} \right)} \right) = 54$ . Then the possible value(s)

of  $a$  is/are :

- (1) 8                                      (2) -9                                      (3) -6                                      (4) 7

**Ans. (1,2)**

3. Three lines

$$L_1 : \vec{r} = \lambda \hat{i}, \lambda \in \mathbb{R},$$

$$L_2 : \vec{r} = \vec{k} + \mu \hat{j}, \mu \in \mathbb{R} \text{ and}$$

$$L_3 : \vec{r} = \hat{i} + \hat{j} + \nu \hat{k}, \nu \in \mathbb{R}$$

are given. For which point(s)  $Q$  and  $L_2$  can we find a point  $P$  on  $L_1$  and a point  $R$  on  $L_3$  so that  $P$ ,  $Q$  and  $R$  are collinear ?

- (1)  $\hat{k} + \hat{j}$                                       (2)  $\hat{k}$                                       (3)  $\hat{k} + \frac{1}{2} \hat{j}$                                       (4)  $\hat{k} - \frac{1}{2} \hat{j}$

**Ans. (2,4)**

4. Let  $F : \mathbb{R} \rightarrow \mathbb{R}$  be a function. We say that  $f$  has

PROPERTY 1 if  $\lim_{h \rightarrow 0} \frac{f(h) - f(0)}{\sqrt{|h|}}$  exists and is finite, and

PROPERTY 2 if  $\lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h^2}$  exists and is finite.

Then which of the following options is/are correct ?

- (1)  $f(x) = x|x|$  has PROPERTY 2                                      (2)  $f(x) = x^{2/3}$  has PROPERTY 1  
 (3)  $f(x) = \sin x$  has PROPERTY 2                                      (4)  $f(x) = |x|$  has PROPERTY 1

**Ans. (2,4)**

5. For non-negative integers  $n$ , let

$$f(n) = \frac{\sum_{k=0}^n \sin\left(\frac{k+1}{x+2} \pi\right) \sin\left(\frac{k+2}{n+2} \pi\right)}{\sum_{k=0}^n \sin^2\left(\frac{k+1}{n+2} \pi\right)}$$

Assuming  $\cos^{-1} x$  takes values in  $[0, \pi]$ , which of the following options is/are correct ?

(1)  $\sin(7 \cos^{-1} f(5)) = 0$  (2)  $f(4) = \frac{\sqrt{3}}{2}$

(3)  $\lim_{n \rightarrow \infty} f(n) = \frac{1}{2}$

(4) If  $\alpha = \tan(\cos^{-1} f(6))$ , then  $\alpha^2 + 2\alpha - 1 = 0$

**Ans. (1,2,4)**

6. Let  $P_1 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ,  $P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ ,  $P_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ,  $P_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ ,  $P_5 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ ,

$$P_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } X = \sum_{k=1}^6 P_k \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} P_k^T$$

where  $P_k^T$  denotes the transpose of the matrix  $P_k$ . Then which of the following options is/are correct?

- (1)  $X - 30I$  is an invertible matrix                      (2) The sum of diagonal entries of  $X$  is 18

- (3) If  $X \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ , then  $\alpha = 30$                       (4)  $X$  is a symmetric matrix

**Ans. (2,3,4)**

7. Let  $x \in \mathbb{R}$  and let  $P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ ,  $Q = \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix}$  and  $R = PQP^{-1}$ .

Then which of the following options is/are correct ?

- (1) For  $x = 1$ , there exists a unit vector  $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$  for which  $R \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

- (2) There exists a real number  $x$  such that  $PQ = QP$

- (3)  $\det R = \det \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8$ , for all  $x \in \mathbb{R}$

- (4) For  $x = 0$ , if  $R \begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = 6 \begin{bmatrix} 1 \\ a \\ b \end{bmatrix}$ , then  $a + b = 5$

**Ans. (3,4)**

8. Let  $f(x) = \frac{\sin \pi x}{x^2}$ ,  $x > 0$

Let  $x_1 < x_2 < x_3 < \dots < x_n < \dots$  be all the points of local maximum of  $f$

and  $y_1 < y_2 < y_3 < \dots < y_n < \dots$  be all the points of local minimum of  $f$ .

Then which of the following options is/are correct ?

- (1)  $|x_n - y_n| > 1$  for every  $n$                       (2)  $x_1 < y_1$

- (3)  $x_n \in \left(2n, 2n + \frac{1}{2}\right)$  for every  $n$                       (4)  $x_{n+1} - x_n > 2$  for every  $n$

**Ans. (1,3,4)**

**SECTION-2 : (Maximum Marks: 18)**

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **Two** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

*Full Marks* : +3 If **ONLY** the correct numerical value is entered.

*Zero Marks* : 0 In all other cases.

1. The value of  $\sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\sec\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)\right)$  in the interval  $\left[-\frac{\pi}{4},\frac{3\pi}{4}\right]$  equals

**Ans. (0.00)**

2. Let  $|X|$  denote the number of elements in set  $X$ . Let  $S = \{1,2,3,4,5,6\}$  be a sample space, where each element is equally likely to occur. If  $A$  and  $B$  are independent events associated with  $S$ , then the number of ordered pairs  $(A,B)$  such that  $1 \leq |B| < |A|$ , equals

**Ans. (243.00)**

3. Five person  $A,B,C,D$  and  $E$  are seated in a circular arrangement. If each of them is given a hat of one of the three colours red, blue and green, then the number of ways of distributing the hats such that the persons seated in adjacent seats get different coloured hats is

**Ans. (30.00)**

4. Suppose

$$\det \begin{bmatrix} \sum_{k=0}^n k & \sum_{k=0}^n {}^n C_k k^2 \\ \sum_{k=0}^n {}^n C_k k & \sum_{k=0}^n {}^n C_k 3^k \end{bmatrix} = 0, \text{ holds for some positive integer } n. \text{ Then } \sum_{k=0}^n \frac{{}^n C_k}{k+1} \text{ equals}$$

**Ans. (6.20)**

5. The value of the integral  $\int_0^{\pi/2} \frac{3\sqrt{\cos\theta}}{(\sqrt{\cos\theta} + \sqrt{\sin\theta})^5} d\theta$  equals

**Ans. (0.15)**

6. Let  $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$  and  $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$  be two vectors. Consider a vector  $\vec{c} = \alpha\vec{a} + \beta\vec{b}$ ,  $\alpha, \beta \in \mathbb{R}$ . If the projection of  $\vec{c}$  on the vector  $(\vec{a} + \vec{b})$  is  $3\sqrt{2}$ , then the minimum value of  $(\vec{c} - (\vec{a} \times \vec{b})) \cdot \vec{c}$  equals

**Ans. (18.00)**

**SECTION-3 : (Maximum Marks : 12)**

- This section contains **TWO (02)** List-Match sets.
- Each List-Match set has **Two (02)** Multiple Choice Questions.
- Each List-Match set has two lists : **List-I** and **List-II**
- **List-I** has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Six** entries (P), (Q), (R), (S), (T) and (U)
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme :

*Full Marks* : +3 If **ONLY** the option corresponding to the correct combination is chosen.

*Zero Marks* : 0 If none of the options is chosen (i.e., the question is unanswered);

*Negative Marks* : -1 In all other cases

**1. Answer the following by appropriately matching the lists based on the information given in the paragraph**

Let  $f(x) = \sin(\pi \cos x)$  and  $g(x) = \cos(2\pi \sin x)$  be two functions defined for  $x > 0$ . Define the following sets whose elements are written in the increasing order :

$$X = \{x : f(x) = 0\}, \quad Y = \{x : f'(x) = 0\}$$

$$Z = \{x : g(x) = 0\}, \quad W = \{x : g'(x) = 0\}.$$

List-I contains the sets X, Y, Z and W. List -II contains some information regarding these sets.

List-I	List-II
(I) X	(P) $\cong \left\{ \frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi \right\}$
(II) Y	(Q) an arithmetic progression
(III) Z	(R) NOT an arithmetic progression
(IV) W	(S) $\cong \left\{ \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6} \right\}$
	(T) $\cong \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \pi \right\}$
	(U) $\cong \left\{ \frac{\pi}{6}, \frac{3\pi}{4} \right\}$

Which of the following is the only correct combination ?

**Options**

(1) (II), (R), (S)

(2) (I), (P), (R)

(3) (II), (Q), (T)

(4) (I), (Q), (U)

**Ans. (3)**

2. Answer the following by appropriately matching the lists based on the information given in the paragraph

Let  $f(x) = \sin(\pi \cos x)$  and  $g(x) = \cos(2\pi \sin x)$  be two functions defined for  $x > 0$ . Define the following sets whose elements are written in the increasing order :

$$X = \{x : f(x) = 0\}, \quad Y = \{x : f'(x) = 0\},$$

$$Z = \{x : g(x) = 0\}, \quad W = \{x : g'(x) = 0\}.$$

List-I contains the sets X, Y, Z and W. List -II contains some information regarding these sets.

List-I	List-II
(I) X	(P) $\supseteq \left\{ \frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi \right\}$
(II) Y	(Q) an arithmetic progression
(III) Z	(R) NOT an arithmetic progression
(IV) W	(S) $\supseteq \left\{ \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6} \right\}$
	(T) $\supseteq \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \pi \right\}$
	(U) $\supseteq \left\{ \frac{\pi}{6}, \frac{3\pi}{4} \right\}$

Which of the following is the only correct combination ?

Options

- (1) (IV), (Q), (T)      (2) (IV), (P), (R), (S)      (3) (III), (R), (U)      (4) (III), (P), (Q), (U)

Ans. (2)

3. Answer the following by appropriately matching the lists based on the information given in the paragraph

Let the circles  $C_1 : x^2 + y^2 = 9$  and  $C_2 : (x-3)^2 + (y-4)^2 = 16$ , intersect at the points X and Y. Suppose that another circle  $C_3 : (x-h)^2 + (y-k)^2 = r^2$  satisfies the following conditions :

(i) centre of  $C_3$  is collinear with the centres of  $C_1$  and  $C_2$

(ii)  $C_1$  and  $C_2$  both lie inside  $C_3$ , and

(iii)  $C_3$  touches  $C_1$  at M and  $C_2$  at N.

Let the line through X and Y intersect  $C_3$  at Z and W, and let a common tangent of  $C_1$  and  $C_3$  be a tangent to the parabola  $x^2 = 8\alpha y$ .

There are some expression given in the List-I whose values are given in List-II below:

List-I	List-II
(I) $2h + k$	(P) 6
(II) $\frac{\text{Length of ZW}}{\text{Length of XY}}$	(Q) $\sqrt{6}$
(III) $\frac{\text{Area of triangle MZN}}{\text{Area of triangle ZMW}}$	(R) $\frac{5}{4}$
(IV) $\alpha$	(S) $\frac{21}{5}$
	(T) $2\sqrt{6}$
	(U) $\frac{10}{3}$

Which of the following is the only INCORRECT combination ?

Options

- (1) (IV), (S)      (2) (IV), (U)      (3) (III), (R)      (4) (I), (P)

Ans. (1)

4. Answer the following by appropriately matching the lists based on the information given in the paragraph

Let the circles  $C_1 : x^2 + y^2 = 9$  and  $C_2 : (x-3)^2 + (y-4)^2 = 16$ , intersect at the points X and Y. Suppose that another circle  $C_3 : (x-h)^2 + (y-k)^2 = r^2$  satisfies the following conditions :

- (i) centre of  $C_3$  is collinear with the centres of  $C_1$  and  $C_2$
- (ii)  $C_1$  and  $C_2$  both lie inside  $C_3$ , and
- (iii)  $C_3$  touches  $C_1$  at M and  $C_2$  at N.

Let the line through X and Y intersect  $C_3$  at Z and W, and let a common tangent of  $C_1$  and  $C_3$  be a tangent to the parabola  $x^2 = 8\alpha y$ .

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(I) $2h + k$	(P) 6
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(IV) $\alpha$	(S) $\frac{21}{5}$
	(T) $2\sqrt{6}$
	(U) $\frac{10}{3}$

Which of the following is the only INCORRECT combination ?

**Options**

- (1) (II), (T)
- (2) (I), (S)
- (3) (I), (U)
- (4) (II), (Q)

**Ans. (4)**