

CAREERS 360

HPBOSE 12TH MATHEMATICS

MODEL QUESTION PAPER

Model Question Paper

Mathematics

Class 10+2

Time Allowed : 3 hours

Max. Marks 85

Special Instructions :-

- (i) Write Question paper series in the circle at the top left side of title page of Answer Book.
 - (ii) While answering questions, indicate on the Answer-Book the same Question No. as appears in the question paper.
 - (iii) Try to answer the questions in serial order as far as possible.
 - (iv) All questions are compulsory.
 - (v) Internal choices have been provided in some questions. Attempt only one of the choices in such questions.
 - (vi) Question Nos. 1 to 10 are multiple choice questions of 1 mark each. Question no. 11 to 13 are of 3 marks each, 14 to 22 are of 4 marks each, 23 to 27 are of 6 marks each.
 - (vii) Use of calculator is not allowed.
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Q1. $\cos^{-1} \left(\cos \frac{7\pi}{6} \right)$ is equal to

- (a) $\frac{7\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$ (1)

Q2. The number of all possible matrices of order 3×3 with each entry 0 or 1 is

- (a) 27 (b) 18 (c) 81 (d) 512 (1)

Q3. The derivative of $\sin^{-1} x$ is

- (a) $\frac{1}{\sqrt{1-x^2}}$ (b) $-\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{1-x^2}$ (d) $\frac{1}{1+x^2}$ (1)

Q4. The approximate change in volume V of a cube of side x metres caused by increasing the side by 2% is : (1)

- (a) $0.06 x^3 m^3$ (b) $0.002 x^3 m^3$ (c) $0.6 x^3 m^3$ (d) $0.006 x^3 m^3$

Q5. $\int \sec x \, dx = ?$

- (a) $\tan x + c$ (b) $\frac{\log}{\sec x} + \frac{\tan x}{+c}$ (c) $\frac{\log}{\sec x} - \tan x$ (d) $\cot x + C$ (1)

(1)

Q6. The degree of the differential equation $\left(\frac{d^2y}{dx^3}\right)^3 + \left(\frac{dy}{dx}\right)^3 + 2y = 0$ is (1)

- (a) 3 (b) 1 (c) 2 (d) Not defined

Q7. Let \vec{a} and \vec{b} be the two unit vectors and θ is the angle between them. Then $\vec{a} + \vec{b}$ is a unit vector if (1)

- (A) $\theta = \frac{\pi}{4}$ (B) $\theta = \frac{\pi}{3}$ (C) $\theta = \frac{\pi}{2}$ (D) $\theta = \frac{2\pi}{3}$

Q8. $\vec{a} \cdot \vec{a} = ?$ (1)

- (A) \vec{a}^2 (B) $|\vec{a}|^2$ (C) 0 (D) $|\vec{a}^2|$

Q9. Distance between two planes $2x+3y+4z = 4$ and $4x+6y+8z=12$ is (1)

- (A) 2 units (B) 4 units (C) 8 units (D) $\frac{2}{\sqrt{29}}$ units

Q10. The probability of obtaining an even prime number on each die when a pair of dice is rolled is : (1)

- (A) 0 (B) $\frac{1}{3}$ (C) $\frac{1}{12}$ (D) $\frac{1}{36}$

Q11. Find all points of discontinuity of f , where f is defined by (3)

$$f(x) = \begin{cases} \frac{1 \times 1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Q12. Find the intervals in which the function f , given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is strictly increasing. (3)

Q13. A die is tossed thrice. Find the probability of getting an odd number atleast once. (3)

Q14. Find $g \circ f$ and $f \circ g$, if $f(x) = |x|$ and $g(x) = |5x - 2|$. (4)

Q15. Prove that: $\cos^{-1} \frac{4}{5} + \cos^{-1} \frac{12}{13} = \cos^{-1} \frac{33}{65}$ (4)

Q16. Express the following matrix as sum of a symmetric and skew symmetric matrix.

$$\begin{pmatrix} 3 & 5 \\ 1 & -1 \end{pmatrix}$$

OR

(2)

By using properties of determinants show that

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a) \quad (4)$$

Q17. Differentiate $\sin(\cos x^2)$ w.r.t. x (4)

Q18. Evaluate: $\int \sqrt{x^2 + 4x + 1} \, dx$ (4)

OR

Evaluate: $\int \frac{3x-1}{(x+2)^2} \, dx$

Q19. Using properties of definite integral evaluate $\int_0^{\frac{\pi}{4}} \log(1 + \tan x) \, dx$ (4)

Q20. Find the general solution of differential equation :

$$x \frac{dy}{dx} + 2y = x^2 \quad (x \neq 0) \quad (4)$$

OR

Solve the differential equation :

$$(1 + e^{\frac{x}{y}}) \, dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) \, dy = 0$$

Q21. Show that the four points A, B, C and D with position vectors $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $-4\hat{i} + 4\hat{j} + 4\hat{k}$ respectively are coplanar. (4)

Q22. Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as “number greater than 4”. (4)

Q23. Solve the system of biner equations, using matrix method : (6)

$$\begin{aligned} 2x + y + z &= 1 \\ x - 2y - z &= \frac{3}{2} \\ 3y - 5z &= 9 \end{aligned}$$

Q24. Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (6)

OR

Using integration, find the area bounded by the curve $|x| + |y| = 1$

Q25. Find the shortest distance between the lines

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda (\hat{i} - 3\hat{j} + 2\hat{k})$$

$$\text{and } \vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu (2\hat{i} + 3\hat{j} + \hat{k})$$

OR

Find the angle between the line

$$\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$$

$$\text{and the plane } 10x + 2y - 11z = 3$$

(6)

Q26. Find the absolute maximum value and absolute minimum value of the function.

$$f(x) = x^3 \text{ in the interval } [-2, 2]$$

OR

Find the equations of tangent and normal to the curve

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2 \text{ at } (1, 1)$$

(6)

Q27. Minimize $z = -3x + 4y$ subject to

$$x + 2y \leq 8$$

$$3x + 2y \leq 12$$

$$x, y \geq 0 \text{ graphically.}$$

(6)

Distribution of Marks

Unit I (9 marks)

1. Relations and functions. 4
2. Inverse trigonometric function $1+4=5$

Unit II (11 marks)

Matrices and determinants $1+4+6 = 11$

Unit III (38 marks)

1. Continuity and differentiability $1+3+4=8$
2. Applications of Derivatives $1+3+6 = 10$
3. Integrals $1+4+9$
4. Application of integrals $6 = 6$
5. Differential equations $1+4 = 5$

Unit IV (13 marks)

1. Vectors $1+1+4 = 6$
2. 3-D Geometry $1+6 = 7$

Unit V (6 marks) Linear Programming 6

Unit VI (8 marks) Probability $1+3+4=8$

Note : (1) Total No. of Questions = 27

(2) Q.No. 1 to 10 of 1 mark each, 11 to 13 are of 3 marks each, 14 to 22 are of 4 marks each, 23 to 27 are of 6 marks each.

Choice :

Q.No. 16,	18,	20,	24,	25,	26	
Marks 4,	4,	4,	6,	6,	6	= 30 marks

Solutions Set
Mathematics 10+2

Q1. $\cos^{-1}\left(\cos \frac{7\pi}{6}\right)$ is equal to

- (a) $\frac{7\pi}{6}$ (b) $\frac{5\pi}{6}$ © $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

Solution 1 : Answer is $\frac{5\pi}{6}$ i.e. b (1)

Q2. The number of all possible matrices of order 3×3 with each entry 0 or 1 is

- (a) 27 (b) 18 (c) 81 (d) 512

Solution 2 : Answer is 512 i.e. d (1)

Q3. The derivative of $\sin^{-1} x$ is

- (a) $\frac{1}{\sqrt{1-x^2}}$ (b) $-\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{1-x^2}$ (d) $\frac{1}{1+x^2}$

Solution 3: Answer is $\frac{1}{\sqrt{1-x^2}}$ i.e. a (1)

Q4. The approximate change in volume V of a cube of side x metres caused by increasing the side by 2% is :

- (a) $0.06 x^3 m^3$ (b) $0.002 x^3 m^3$ (c) $0.6 x^3 m^3$ (d) $0.006 x^3 m^3$

Solution 4 : Answer is $0.06 x^3 m^3$ i.e. a (1)

Q5. $\int \sec x \, dx = ?$

- (a) $\tan x + c$ (b) $\frac{\log}{\sec x} + \frac{\tan x}{+c}$ (c) $\frac{\log}{\sec x} - \tan x$ (d) $\cot x + C$

Solution 5 : Answer is $\frac{\log}{\sec x} + \frac{\tan x}{+c}$ i.e. b (1)

Q6. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^3 + 2y = 0$ is

- (a) 3 (b) 1 © 2 (d) Not defined (1)

Solution 6 : Answer is 3 i.e. a

(1)

Q7. Let \vec{a} and \vec{b} be the two unit vectors and θ is the angle between them. Then $\vec{a} + \vec{b}$ is a unit vector if

- (A) $\theta = \frac{\pi}{4}$ (B) $\theta = \frac{\pi}{3}$ (C) $\theta = \frac{\pi}{2}$ (D) $\theta = \frac{2\pi}{3}$ (1)

Solution 7 : Answer is $\theta = \frac{2\pi}{3}$ i.e. D

Q8. $\vec{a} \cdot \vec{a} = ?$ (1)

- (A) \vec{a}^2 (B) $|\vec{a}|^2$ (C) 0 (D) $|\vec{a}^2|$

Solution 8 : Answer is $|\vec{a}|^2$ i.e. B

Q9. Distance between two planes $2x + 3y + 4z = 4$ and $4x + 6y + 8z = 12$ (1)

- (A) 2 units (B) 4 units (C) 8 units (D) $\frac{2}{\sqrt{29}}$ units

Solution 9 : Answer is $\frac{2}{\sqrt{29}}$ units i.e. D.

Q10. The probability of obtaining an even prime number on each die when a pair of dice is rolled is : (1)

- (A) 0 (B) $\frac{1}{3}$ (C) $\frac{1}{12}$ (D) $\frac{1}{36}$

Solution 10 : Answer is $\frac{1}{36}$ i.e. D

Q11. Find all points of discontinuity of f , where f is defined by (3)

$$f(x) = \begin{cases} \frac{1 \times 1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Solution 11 : L.H.L. = $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{(-x)}{x} = -1$

$$\text{R.H.L.} = \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{x}{x} = 1$$

L.H.L. \neq R.H.L.

$\therefore f$ is discontinuous at $x = 0$

Q12. Find the intervals in which the function f , given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is strictly increasing. (3)

Solution 12: Let $f'(x) = 0$
 $\Rightarrow 6(x-3)(x+2) = 0$
 $x = 3$ or -2
 f is strictly increasing in $(-\infty, -2) \cup (3, \infty)$

Q13. A die is tossed thrice. Find the probability of getting an odd number atleast once. (3)

Solution 13: $P(A) = \frac{3}{6} = \frac{1}{2}$
 Required probability = $P(\text{atleast an odd number})$
 $= 1 - P(\bar{A} \bar{A} \bar{A})$
 $= 1 - \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{2}\right)$
 $= \frac{7}{8}$

Q14. Find $g \circ f$ and $f \circ g$, if $f(x) = |x|$ and $g(x) = |5x - 2|$. (4)

Solution 14: $f(x) = |x|$, $g(x) = |5x - 2|$
 $(g \circ f)(x) = g(f(x)) = g(|x|) = |5|x| - 2|$
 $(f \circ g)(x) = f(g(x)) = f(|5x - 2|) = ||5x - 2||$

Q15. Prove that: $\cos^{-1} \frac{4}{5} + \cos^{-1} \frac{12}{13} = \cos^{-1} \frac{33}{65}$

Solution 15: Let $\cos^{-1} \frac{4}{5} = x$ and $\cos^{-1} \frac{12}{13} = y$

We know that

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$= \frac{4}{5} \times \frac{12}{13} - \frac{3}{5} \times \frac{5}{13}$$

$$= \frac{33}{65}$$

$$x + y = \cos^{-1} \frac{33}{65}$$

$$\cos^{-1} \frac{4}{5} + \cos^{-1} \frac{12}{13} = \cos^{-1} \frac{33}{65}$$

(3)

Q16. Express the following matrix as sum of a symmetric and skew symmetric matrix.

$$\begin{pmatrix} 3 & 5 \\ 1 & -1 \end{pmatrix}$$

Solution 16: $A = \begin{pmatrix} 3 & 5 \\ 1 & -1 \end{pmatrix}$ and $A' = \begin{pmatrix} 3 & 1 \\ 5 & -1 \end{pmatrix}$

$$P = \frac{1}{2}(A+A')$$

$$= \frac{1}{2} \begin{pmatrix} 6 & 6 \\ 6 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 3 \\ 3 & -1 \end{pmatrix}$$

$$\Rightarrow P' = P$$

$\Rightarrow P$ is symmetric matrix.

$$Q = \frac{1}{2}(A-A')$$

$$Q = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}$$

$$\Rightarrow Q' = -Q$$

$$\Rightarrow P+Q = \begin{pmatrix} 3 & 3 \\ 3 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ 1 & -1 \end{pmatrix} = A$$

A is sum of symmetric & skew symmetric Matrix.

OR

By using properties of determinants show that

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

Solution 16 :

$$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Operate $R_2 \rightarrow R_2 - R_1$
 $R_3 \rightarrow R_3 - R_1$

$$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$$

Taking common & expanding

$$\Delta = (a-b)(b-c)(c-a) \quad (4)$$

Q17. Differentiate $\sin(\cos x^2)$ w.r.t. x

Solution 17: Let $y = \sin(\cos(x^2))$

$$\begin{aligned} \frac{dy}{dx} &= \cos(\cos x^2) (-\sin x^2) \frac{d}{dx} x^2 \\ &= -2x \sin x^2 \cos(\cos x^2) \end{aligned}$$

Q18. Evaluate: $\int \sqrt{x^2+4x+1} dx$ (4)

Solution 18: Let $I = \int \sqrt{x^2+4x+1} dx$

$$= \int \sqrt{(x+2)^2 - (\sqrt{3})^2} dx \quad \text{using formula } \int \sqrt{x^2-a^2}$$

$$= \frac{(x+2)}{2} \sqrt{(x+2)^2 - (\sqrt{3})^2} - \frac{3}{2} \log [(x+2) + \sqrt{(x+2)^2 - (\sqrt{3})^2}] + C$$

$$= \frac{(x+2)}{2} \sqrt{x^2+4x+1} - \frac{3}{2} \log [(x+2) + \sqrt{x^2+4x+1}] + C$$

OR

Evaluate: $\int \frac{3x-1}{(x+2)^2} dx$

Solution 18: Let $I = \int \frac{3x-1}{(x-2)^2} dx$

$$\frac{3x-1}{(x-2)^2} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2}$$

$$I = \int \frac{3x-1}{(x-2)^2} dx = 3 \int \frac{dx}{(x-2)} + 5 \int \frac{dx}{(x-2)^2}$$

$$= 3 \log(x-2) - 5 \left(\frac{1}{(x-2)} \right) + C$$

(5)

Q19. Using properties of definite integral evaluate $\int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx$ (4)

Solution19 : Let $I = \int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx$

Using $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

We get

$$I = \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1 + \tan x}\right) dx$$

$$2I = \int_0^{\frac{\pi}{4}} \log 2 dx$$

$$I = \frac{1}{2} \log 2 [x]_0^{\frac{\pi}{4}}$$

$$I = \frac{\pi}{8} \log 2$$

Q20. Find the general solution of differential equation :

$$x \frac{dy}{dx} + 2y = x^2 \quad (x \neq 0)$$

Solution 20 : Given differential equation is

$$x \frac{dy}{dx} + 2y = x^2$$

$$\frac{dy}{dx} + \frac{2}{x} y = x$$

$$\text{I.F.} = e^{\int \frac{2}{x} dx} = x^2$$

$$y \cdot x^2 = \int x^3 dx + c$$

$$y = \frac{x^2}{4} + cx^{-2}$$

OR

(6)

Solve the differential equation :

$$(1 + e^{\frac{x}{y}}) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$$

OR

$$\text{Solution : } \frac{dx}{dy} = \frac{e^{-\frac{x}{y}} \left(1 - \frac{x}{y}\right)}{1 + e^{\frac{x}{y}}}$$

$$\text{Put } x = vy \Rightarrow \frac{dx}{dy} = v + y \frac{dv}{dy}$$

$$v + y \frac{dv}{dy} = \frac{-e^v(1-v)}{1 + e^v}$$

$$y \frac{dv}{dy} = \frac{-(e^v + v)}{e^v + 1}$$

Integrating we get

$$\log(e^v + v)y = \log c$$

$$-x + ye^{\frac{x}{y}} = c$$

- Q21. Show that the four points A, B, C and D with position vectors $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $-4\hat{i} + 4\hat{j} + 4\hat{k}$ respectively are coplanar. (4)

Solution 21 :

$$[AB \quad AC \quad AD] = \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = 0$$

Hence A, B, C and D are coplanar.

- Q22. Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as "number greater than 4".

Solution 22 : Sample space is {1, 2, 3, 4, 5, 6}

$$P(\text{Success}) = \frac{1}{3}$$

$$x = 0, 1, 2$$

$$P(x=0) = P(\bar{S} \bar{S}) = \frac{4}{9}$$

$$P(x=1) = P(S \bar{S} \text{ or } \bar{S} S) = \frac{4}{9}$$

(7)

$$P(x=2) = P(SS) = \frac{1}{9}$$

Probability distribution

x	0	1	2
P(x)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

Q23. Solve the system of binear equations, using matrix method :

(6)

$$2x + y + z = 1$$

$$x - 2y - z = \frac{3}{2}$$

$$3y - 5z = 9$$

Solution 23 : Let $Ax = B$

$$|A| = 34 \neq 0$$

$$\text{Adj}A = \begin{vmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{vmatrix}$$

$$A^{-1} = \frac{\text{Adj}A}{|A|} = \frac{1}{34} \begin{vmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{vmatrix}$$

$$X = \begin{bmatrix} 1 \\ \frac{1}{2} \\ -\frac{3}{2} \end{bmatrix} \Rightarrow x = 1, y = \frac{1}{2}, z = -\frac{3}{2}$$

Q24. Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(6)

Solution 24 : Considering horizontal strips

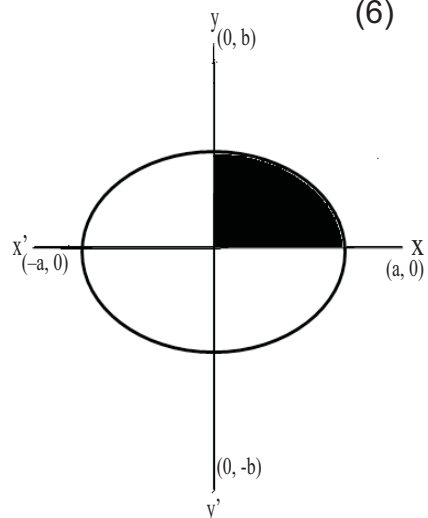
Area of Ellipse

$$= 4 \int_0^b x dy$$

$$= \frac{4a}{b} \left[\frac{y}{2} \sqrt{b^2 - y^2} + \frac{b^2}{2} \sin^{-1} \frac{y}{b} \right]_0^b$$

$$= \frac{4a}{b} \cdot \frac{b^2 \pi}{2} = \pi ab$$

OR



(8)

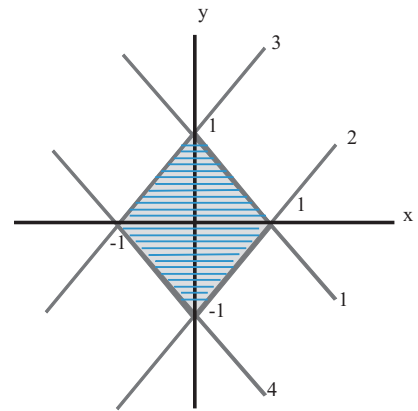
Using integration, find the area bounded by the curve $|x| + |y| = 1$

Solution: Given Curve $|x| + |y| = 1$

$$x + y = 1 \rightarrow (1) \quad x - y = 1 \rightarrow (2)$$

$$-x + y = 1 \rightarrow (3) \quad -x - y = 1 \rightarrow (4)$$

$$\begin{aligned} \text{Required area} &= 4 \int_{\text{line 1}}^1 y \, dx = 4 \left[x - \frac{x^2}{2} \right]_0^1 \\ &= 4 \left[\frac{1}{2} \right] = 2 \end{aligned}$$



Q25. Find the shortest distance between the lines

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda (\hat{i} - 3\hat{j} + 2\hat{k})$$

$$\text{and } \vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu (2\hat{i} + 3\hat{j} + \hat{k})$$

Solution 25:

$$\text{Here } \vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$$

$$\vec{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}, \vec{b}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = -9\hat{i} + 3\hat{j} + 9\hat{k}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{171}$$

$$\text{S.D.} = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{(\vec{b}_1 \times \vec{b}_2)} \right| = \left| \frac{-27 + 9 + 27}{\sqrt{171}} \right| = \frac{9}{\sqrt{171}} \text{ units}$$

OR

Find the angle between the line

$$\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$$

and the plane $10x + 2y - 11z = 3$

Solution: $\vec{r} = (-\hat{i} + 3\hat{k}) + \lambda (2\hat{i} + 3\hat{j} + 6\hat{k})$

$$\sin \theta = \left| \frac{(2\hat{i} + 3\hat{j} + 6\hat{k}) \cdot (10\hat{i} + 2\hat{j} - 11\hat{k})}{\sqrt{2^2 + 3^2 + 6^2} \cdot \sqrt{10^2 + 2^2 + (11)^2}} \right|$$

(9)

$$= \frac{8}{21}$$

$$\theta = \sin^{-1} \frac{8}{21}$$

Q26. Find the absolute maximum value and absolute minimum value of the function.

$$f(x) = x^3 \text{ in the interval } [-2, 2]$$

Solution 26: $f(x) = x^3, x \in [-2, 2]$

$$f'(x) = 0$$

$$3x^2 = 0$$

$$x = 0 \in [-2, 2]$$

Absolute maximum value of $f(x) = 8$ at $x = 2$

Absolute minimum value of $f(x) = -8$, at $x = -2$

OR

Find the equations of tangent and normal to the curve

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2 \text{ at } (1, 1)$$

Solution 26: Differentiating $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$

$$\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$$

$$\text{Slope of tangent at } (1, 1) \left. \frac{dy}{dx} \right|_{(1,1)} = -1$$

$$\text{Equation of tangent } y + x - 2 = 0$$

$$\text{Equation of normal is } y - x = 0$$

Q27. Minimize $z = -3x + 4y$ subject to

$$x + 2y \leq 8$$

$$3x + 2y \leq 12$$

$$x, y \geq 0 \text{ graphically.}$$

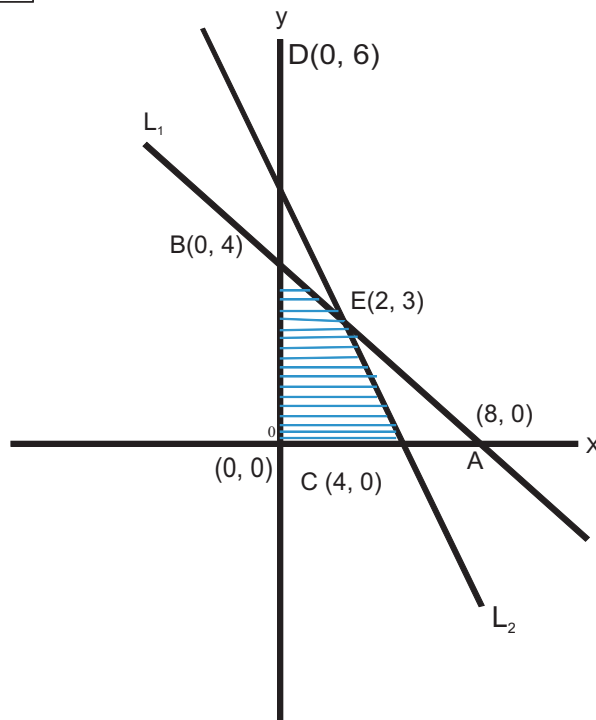
Solution 27 :

$$L_1 : x + 2y = 8$$

x	8	0
y	0	4

$$L_2 : 3x + 2y = 12$$

x	4	0
y	0	6



Corner Points	$z = -3x + 4y$
$O(0, 0)$	0
$B(0, 4)$	16
$C(4, 0)$	-12
$E(2, 3)$	6

→ Minimum

Z is minimum at $C(4, 0)$, $x = 4$, $y = 0$