

Syllabus for Mathematics Paper

Calculus: Finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property; Sequences and series, convergence; Limits, continuity, uniform continuity, differentiability, mean value theorems; Riemann integration, Improper integrals; Functions of two or three variables, continuity, directional derivatives, partial derivatives, total derivative, maxima and minima, saddle point, method of Lagrange's multipliers; Double and Triple integrals and their applications; Line integrals and Surface integrals, Green's theorem, Stokes' theorem, and Gauss divergence theorem.

Linear Algebra: Finite dimensional vector spaces over real or complex fields; Linear transformations and their matrix representations, rank and nullity; systems of linear equations, eigenvalues and eigenvectors, minimal polynomial, Cayley-Hamilton Theorem, diagonalization, Jordan canonical form, symmetric, skew-symmetric, Hermitian, skew-Hermitian, orthogonal and unitary matrices; Finite dimensional inner product spaces, Gram-Schmidt orthonormalization process, definite forms.

Real Analysis: Metric spaces, connectedness, compactness, completeness; Sequences and series of functions, uniform convergence; Weierstrass approximation theorem; Power series; Functions of several variables: Differentiation, contraction mapping principle, Inverse and Implicit function theorems; Lebesgue measure, measurable functions; Lebesgue integral, Fatou's lemma, monotone convergence theorem, dominated convergence theorem.

Complex Analysis: Analytic functions, harmonic functions; Complex integration: Cauchy's integral theorem and formula; Liouville's theorem, maximum modulus principle, Morera's theorem; zeros and singularities; Power series, radius of convergence, Taylor's theorem and Laurent's theorem; residue theorem and applications for evaluating real integrals; Rouché's theorem, Argument principle, Schwarz lemma; conformal mappings, bilinear transformations.

Ordinary Differential equations: First order ordinary differential equations, existence and uniqueness theorems for initial value problems, linear ordinary differential equations of higher order with constant coefficients; Second order linear ordinary differential equations with variable coefficients; Cauchy-Euler equation, method of Laplace transforms for solving ordinary differential equations, series solutions (power series, Frobenius method); Legendre and Bessel functions and their orthogonal properties; Systems of linear first order ordinary differential equations.

Algebra: Groups, subgroups, normal subgroups, quotient groups, homomorphisms, automorphisms; cyclic groups, permutation groups, Sylow's theorems and their applications; Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domains, Principle ideal domains, Euclidean domains, polynomial rings and irreducibility criteria; Fields, finite fields, field extensions.

Functional Analysis: Normed linear spaces, Banach spaces, Hahn-Banach theorem, open mapping and closed graph theorems, principle of uniform boundedness; Inner-product spaces, Hilbert spaces, orthonormal bases, Riesz representation theorem.

Numerical Analysis: Numerical solutions of algebraic and transcendental equations: bisection, secant method, Newton-Raphson method, fixed point iteration; Interpolation: error of polynomial interpolation, Lagrange and Newton interpolations; Numerical differentiation; Numerical integration: Trapezoidal and Simpson's rules; Numerical solution of a system of linear equations: direct methods (Gauss elimination, LU decomposition), iterative methods (Jacobi and Gauss-Seidel); Numerical solution of initial value problems of ODEs: Euler's method, Runge-Kutta methods of order 2.

Partial Differential Equations: Linear and quasi-linear first order partial differential equations, method of characteristics; Second order linear equations in two variables and their classification; Cauchy, Dirichlet and Neumann problems; Solutions of Laplace and wave equations in two dimensional Cartesian coordinates, interior and exterior Dirichlet problems in polar coordinates; Separation of variables method for solving wave and diffusion equations in one space variable; Fourier series and Fourier transform and Laplace transform methods of solutions for the equations mentioned above.

Topology: Basic concepts of topology, bases, subbases, subspace topology, order topology, product topology, metric topology, connectedness, compactness, countability and separation axioms, Urysohn's Lemma.

Linear Programming: Linear programming problem and its formulation, convex sets and their properties, graphical method, basic feasible solution, simplex method, two phase methods; infeasible and unbounded LPP's, alternate optima; Dual problem and duality theorems; Balanced and unbalanced transportation problems, Vogel's approximation method for solving transportation problems; Hungarian method for solving assignment problems.