FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

(Held On Thursday 09th JANUARY, 2020) TIME : 9 : 30 AM to 12 : 30 PM

MATHEMATICS

TEST PAPER WITH ANSWER

1. A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness the melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate (in cm/min.) at which of the thickness of ice decreases, is :

(1) $\frac{1}{36\pi}$ (2) $\frac{5}{6\pi}$ (3) $\frac{1}{18\pi}$ (4) $\frac{1}{54\pi}$

NTA Ans. (3)

ALLEN Ans. (3)

2. If the number of five digit numbers with distinct digits and 2 at the 10th place is 336 k, then k is equal to :

(1) 8 (2) 6 (3) 4 (4) 7

NTA Ans. (1)

ALLEN Ans. (1)

3. Let z be complex number such that $\left|\frac{z-i}{z+2i}\right|=1$ and $|z|=\frac{5}{2}$. Then the value of $|z + 3i|$ is :

(1) $\sqrt{10}$ (2) $2\sqrt{3}$ (3) $\frac{7}{2}$ (4) $\frac{15}{4}$

NTA Ans. (3)

ALLEN Ans. (3)

4. In a box, there are 20 cards, out of which 10 are lebelled as A and the remaining 10 are labelled as B. Cards are drawn at random, one after the other and with replacement, till a second A-card is obtained. The probability that the second A-card appears before the third B-card is :

(1) $\frac{11}{16}$ (2) $\frac{13}{16}$ (3) $\frac{9}{16}$ (4) $\frac{15}{16}$

NTA Ans. (1)

ALLEN Ans. (1)

5. The value of $\int_0^{2\pi}\frac{x\sin^8 x}{\sin^8 x+\cos^8 x}dx$ is equal to :

(1) 2π (2) 4π (3) $2\pi^2$ (4) π^2

NTA Ans. (4)

ALLEN Ans. (4)

6. If $f'(x) = \tan^{-1}(\sec x + \tan x)$, $-\frac{\pi}{2}<x<\frac{\pi}{2}$, and $f(0) = 0$, then $f(1)$ is equal to :

(1) $\frac{\pi-1}{4}$ (2) $\frac{\pi+2}{4}$ (3) $\frac{\pi+1}{4}$ (4) $\frac{1}{4}$

NTA Ans. (3)

ALLEN Ans. (3)

7. If the matrices $A=\begin{bmatrix}1 & 1 & 2\\ 1 & 3 & 4\\ 1 & -1 & 3\end{bmatrix}$, B = adjA and C = 3A, then $\frac{|adjB|}{|C|}$ is equal to :

(1) 72 (2) 2 (3) 8 (4) 16

NTA Ans. (3)

ALLEN Ans. (3)

8. The number of real roots of the equation, $e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$ is :

(1) 4 (2) 2 (3) 3 (4) 1

NTA Ans. (4)

ALLEN Ans. (4)

9. Negation of the statement :

$\sqrt{5}$ is an integer or 5 is irrational is :

(1) $\sqrt{5}$ is irrational or 5 is an integer.

(2) $\sqrt{5}$ is not an integer and 5 is not irrational.

(3) $\sqrt{5}$ is an integer and 5 is irrational.

(4) $\sqrt{5}$ is not an integer or 5 is not irrational.

NTA Ans. (2)

ALLEN Ans. (2)

10. Let the observations $x_i (1 \le i \le 10)$ satisfy the equations, $\sum_{i=1}^{10}(x_i - 5) = 10$ and $\sum_{i=1}^{10}(x_i - 5)^2 = 40$. If μ and λ are the mean and the variance of the observations, $x_1 - 3$, $x_2 - 3$, ..., $x_{10} - 3$, then the ordered pair (μ, λ) is equal to :

(1) (6, 6) (2) (3, 6) (3) (6, 3) (4) (3, 3)

NTA Ans. (4)

ALLEN Ans. (4)

11. The product $2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}} \cdot \ldots$ to ∞ is equal to :

(1) $2^{\frac{1}{2}}$ (2) $2^{\frac{1}{4}}$ (3) 2 (4) 1

NTA Ans. (1)

ALLEN Ans. (1)

12. A circle touches the y-axis at the point (0, 4) and passes through the point (2, 0). Which of the following lines is not a tangent to this circle ?

(1) $3x - 4y - 24 = 0$ (2) $3x + 4y - 6 = 0$

(3) $4x + 3y - 8 = 0$ (4) $4x - 3y + 17 = 0$

NTA Ans. (3)

ALLEN Ans. (3)

13. If e_1 and e_2 are the eccentricities of the ellipse, $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and the hyperbola, $\frac{x^2}{9} - \frac{y^2}{4} = 1$ respectively and (e_1, e_2) is a point on the ellipse, $15x^2 + 3y^2 = k$, then k is equal to :

(1) 15 (2) 14 (3) 17 (4) 16

NTA Ans. (4)

ALLEN Ans. (4)

14. Let f be any function continuous on [a, b] and twice differentiable on (a, b). If for all $x \in (a, b)$, $f'(x) > 0$ and $f''(x) < 0$, then for any $c \in (a, b)$, $\frac{f(c) - f(a)}{f(b) - f(c)}$ is greater than :

(1) $\frac{b+a}{b-a}$ (2) $\frac{b-c}{c-a}$ (3) $\frac{c-a}{b-c}$ (4) 1

NTA Ans. (3)

ALLEN Ans. (3)

15. If for some α and β in R, the intersection of the following three places

$x + 4y - 2z = 1$

$x + 7y - 5z = \beta$

$x + 5y + \alpha z = 5$

is a line in R^3, then $\alpha + \beta$ is equal to :

(1) 10 (2) –10 (3) 2 (4) 0

NTA Ans. (1)

ALLEN Ans. (1)

16. The integral $\int \frac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}}$ is equal to :

(where C is a constant of integration)

(1) $\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + C$

(2) $-\left(\frac{x-3}{x+4}\right)^{-\frac{1}{7}} + C$

(3) $\frac{1}{2}\left(\frac{x-3}{x+4}\right)^{\frac{3}{7}} + C$

(4) $-\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{-\frac{13}{7}} + C$

NTA Ans. (1)

ALLEN Ans. (1)

17. Let C be the centroid of the triangle with vertices (3, –1), (1, 3) and (2, 4). Let P be the point of intersection of the lines $x + 3y - 1 = 0$ and $3x - y + 1 = 0$. Then the line passing through the points C and P also passes through the point :

(1) (7, 6) (2) (–9, –6)

(3) (–9, –7) (4) (9, 7)

NTA Ans. (2)

ALLEN Ans. (2)

18. If $f(x)=\begin{cases}\dfrac{\sin(a+2)x+\sin x}{x} & ; x<0\\ b & ; x=0\\ \dfrac{(x+3x^2)^{\frac{1}{3}}-x^{-\frac{1}{3}}}{x^{\frac{4}{3}}} & ; x>0\end{cases}$

is continuous at x = 0, then a + 2b is equal to :

(1) –1 (2) 1 (3) –2 (4) 0

NTA Ans. (4)

ALLEN Ans. (4)

19. The value of

$$\cos^3\left(\frac{\pi}{8}\right)\cdot\cos\left(\frac{3\pi}{8}\right)+\sin^3\left(\frac{\pi}{8}\right)\cdot\sin\left(\frac{3\pi}{8}\right)$$

is :

(1) $\frac{1}{4}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{2\sqrt{2}}$ (4) $\frac{1}{2}$

NTA Ans. (3)

ALLEN Ans. (3)

20. If for all real triplets (a, b, c), $f(x) = a + bx + cx^2$;

then $\int_0^1 f(x)dx$ is equal to :

(1) $\frac{1}{2}\left\{f(1)+3f\left(\frac{1}{2}\right)\right\}$

(2) $2\left\{3f(1)+2f\left(\frac{1}{2}\right)\right\}$

(3) $\frac{1}{6}\left\{f(0)+f(1)+4f\left(\frac{1}{2}\right)\right\}$

(4) $\frac{1}{3}\left\{f(0)+f\left(\frac{1}{2}\right)\right\}$

NTA Ans. (3)

ALLEN Ans. (3)

21. The coefficient of x^4 is the expansion of $(1 + x + x^2)^{10}$ is ———.

NTA Ans. (615.00)

ALLEN Ans. (615.00)

22. The number of distinct solutions of the equation $\log_{\frac{1}{2}}|\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|$ in the interval $[0, 2\pi]$, is ———.

NTA Ans. (8.00)

ALLEN Ans. (8.00)

23. If for $x \geq 0$, $y = y(x)$ is the solution of the differential equation

$(x + 1)dy = ((x + 1)^2 + y - 3)dx$, $y(2) = 0$,

then $y(3)$ is equal to ———.

NTA Ans. (3.00)

ALLEN Ans. (3.00)

24. If the vectors, $\vec{p}=(a+1)\hat{i}+a\hat{j}+a\hat{k}$,

$\vec{q}=a\hat{i}+(a+1)\hat{j}+a\hat{k}$ and

$\vec{r}=a\hat{i}+a\hat{j}+(a+1)\hat{k}$ $(a\in R)$ are coplanar and $3(\vec{p}.\vec{q})^2-\lambda|\vec{r}\times\vec{q}|^2=0$, then the value of λ is ———.

NTA Ans. (1.00)

ALLEN Ans. (1.00)

25. The projection of the line segment joining the points (1, –1, 3) and (2, –4, 11) on the line joining the points (–1, 2, 3) and (3, –2, 10) is ———.

NTA Ans. (8.00)

ALLEN Ans. (8.00)