FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

(Held On Thursday 09th JANUARY, 2020) TIME : 2 : 30 PM to 5 : 30 PM

MATHEMATICS

TEST PAPER WITH ANSWER

1. Let [t] denote the greatest integer ≤ t and $\lim\limits_{x\to 0} x\left[\frac{4}{x}\right] = A$. Then the function, $f(x) = [x^2]\sin(\pi x)$ is discontinuous, when x is equal to :

(1) $\sqrt{A+5}$

(2) $\sqrt{A+1}$

(3) $\sqrt{A}$

(4) $\sqrt{A+21}$

NTA Ans. (2)

ALLEN Ans. (2)

2. The following system of linear equations

$7x + 6y - 2z = 0$

$3x + 4y + 2z = 0$

$x - 2y - 6z = 0$, has

(1) infinitely many solutions, (x, y, z) satisfying $x = 2z$

(2) no solution

(3) only the trivial solution

(4) infinitely many solutions, (x, y, z) satisfying $y = 2z$

NTA Ans. (1)

ALLEN Ans. (1)

3. If $x = 2\sin\theta - \sin 2\theta$ and $y = 2\cos\theta - \cos 2\theta$, $\theta \in [0, 2\pi]$, then $\frac{d^2y}{dx^2}$ at $\theta = \pi$ is :

(1) $\frac{3}{2}$ (2) $-\frac{3}{4}$

(3) $\frac{3}{4}$ (4) $-\frac{3}{8}$

NTA Ans. (4)

ALLEN Ans. (Bonus)

4. The length of the minor axis (along y-axis) of an ellipse in the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the line, $x + 6y = 8$; then its eccentricity is :

(1) $\sqrt{\frac{5}{6}}$ (2) $\frac{1}{2}\sqrt{\frac{11}{3}}$ (3) $\frac{1}{3}\sqrt{\frac{11}{3}}$ (4) $\frac{1}{2}\sqrt{\frac{5}{3}}$

NTA Ans. (2)

ALLEN Ans. (2)

5. Let $a, b \in R$, $a \neq 0$ be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to :

(1) 26 (2) 25 (3) 28 (4) 24

NTA Ans. (2)

ALLEN Ans. (2)

6. Given : $f(x) = \begin{cases} x & , \ 0 \le x < \frac{1}{2} \\ \frac{1}{2} & , \ x = \frac{1}{2} \\ 1-x & , \ \frac{1}{2} < x \le 1 \end{cases}$

and $g(x) = \left(x - \frac{1}{2}\right)^2$, $x \in R$. Then the area (in sq. units) of the region bounded by the curves, $y = f(x)$ and $y = g(x)$ between the lines, $2x = 1$ and $2x = \sqrt{3}$, is :

(1) $\frac{1}{3} + \frac{\sqrt{3}}{4}$ (2) $\frac{\sqrt{3}}{4} - \frac{1}{3}$

(3) $\frac{1}{2} + \frac{\sqrt{3}}{4}$ (4) $\frac{1}{2} - \frac{\sqrt{3}}{4}$

NTA Ans. (2)

ALLEN Ans. (2)

7. A random variable X has the following probability distribution :

X	:	1	2	3	4	5
P(X)	:	K^2	2K	K	2K	$5K^2$

Then P(X > 2) is equal to :

(1) $\frac{7}{12}$ (2) $\frac{23}{36}$ (3) $\frac{1}{36}$ (4) $\frac{1}{6}$

NTA Ans. (2)

ALLEN Ans. (2)

8. If $x=\sum_{n=0}^{\infty}(-1)^n\tan^{2n}\theta$ and $y=\sum_{n=0}^{\infty}\cos^{2n}\theta$, for $0<\theta<\frac{\pi}{4}$, then :

(1) y(1 + x) = 1 (2) x(1 + y) = 1

(3) y(1 − x) = 1 (4) x(1 − y) = 1

NTA Ans. (3)

ALLEN Ans. (3)

9. Let a function $f : [0, 5] \to \mathbf{R}$ be continuous, $f(1) = 3$ and F be defined as :

$F(x)=\int_1^x t^2g(t)\,dt$, where $g(t)=\int_1^t f(u)\,du$.

Then for the function F, the point x = 1 is :

(1) a point of local minima.
(2) not a critical point.
(3) a point of inflection.
(4) a point of local maxima.

NTA Ans. (1)

ALLEN Ans. (1)

10. If one end of a focal chord AB of the parabola $y^2 = 8x$ is at $A\left(\frac{1}{2},-2\right)$, then the equation of the tangent to it at B is :

(1) 2x + y − 24 = 0 (2) x − 2y + 8 = 0

(3) 2x − y − 24 = 0 (4) x + 2y + 8 = 0

NTA Ans. (2)

ALLEN Ans. (2)

11. If 10 different balls are to be placed in 4 distinct boxes at random, then the probability that two of these boxes contain exactly 2 and 3 balls is :

(1) $\frac{945}{2^{11}}$ (2) $\frac{965}{2^{11}}$ (3) $\frac{945}{2^{10}}$ (4) $\frac{965}{2^{10}}$

NTA Ans. (3)

ALLEN Ans. (3)

12. If $A = \{x \in \mathbf{R} : |x| < 2\}$ and $B = \{x \in \mathbf{R} : |x - 2| \geq 3\}$; then :

(1) $A \cup B = \mathbf{R} - (2, 5)$

(2) $A \cap B = (-2, -1)$

(3) $B - A = \mathbf{R} - (-2, 5)$

(4) $A - B = [-1, 2)$

NTA Ans. (3)

ALLEN Ans. (3)

13. If $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$; y(1) = 1; then a value of x satisfying y(x) = e is :

(1) $\sqrt{2}e$ (2) $\frac{e}{\sqrt{2}}$ (3) $\frac{1}{2}\sqrt{3}e$ (4) $\sqrt{3}e$

NTA Ans. (4)

ALLEN Ans. (4)

14. If $\int\frac{d\theta}{\cos^2\theta(\tan 2\theta+\sec 2\theta)}=\lambda\tan\theta+2\log_e|f(\theta)|+C$

where C is a constant of integration, then the ordered pair $(\lambda, f(\theta))$ is equal to :

(1) (−1, 1 + tanθ) (2) (−1, 1 − tanθ)

(3) (1, 1 − tanθ) (4) (1, 1 + tanθ)

NTA Ans. (1)

ALLEN Ans. (1)

15. If z be a complex number satisfying |Re(z)| + |Im(z)| = 4, then |z| cannot be

(1) $\sqrt{\frac{17}{2}}$ (2) $\sqrt{10}$ (3) $\sqrt{8}$ (4) $\sqrt{7}$

NTA Ans. (4)

ALLEN Ans. (4)

16. If $p \rightarrow (p \wedge \sim q)$ is false, then the truth values of p and q are respectively :

(1) F, T (2) T, T (3) F, F (4) T, F

NTA Ans. (2)

ALLEN Ans. (2)

17. Let $a - 2b + c = 1$.

If $f(x)=\begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$, then :

(1) $f(-50) = 501$ (2) $f(-50) = -1$

(3) $f(50) = 1$ (4) $f(50) = -501$

NTA Ans. (3)

ALLEN Ans. (3)

18. In the expansion of $\left(\frac{x}{\cos\theta}+\frac{1}{x\sin\theta}\right)^{16}$, if ℓ_1 is the least value of the term independent of x when $\frac{\pi}{8}\le\theta\le\frac{\pi}{4}$ and ℓ_2 is the least value of the term independent of x when $\frac{\pi}{16}\le\theta\le\frac{\pi}{8}$, then the ratio $\ell_2 : \ell_1$ is equal to :

(1) 1 : 8 (2) 1 : 16 (3) 8 : 1 (4) 16 : 1

NTA Ans. (4)

ALLEN Ans. (4)

19. Let a_n be the n^{th} term of a G.P. of positive terms.

If $\sum_{n=1}^{100} a_{2n+1} = 200$ and $\sum_{n=1}^{100} a_{2n} = 100$, then $\sum_{n=1}^{200} a_n$ is equal to :

(1) 225 (2) 175 (3) 300 (4) 150

NTA Ans. (4)

ALLEN Ans. (4)

20. Let f and g be differentiable functions on **R** such that fog is the identity function. If for some a, b $\in$ **R**, g'(a) = 5 and g(a) = b, then $f'(b)$ is equal to :

(1) $\frac{2}{5}$ (2) 1 (3) $\frac{1}{5}$ (4) 5

NTA Ans. (3)

ALLEN Ans. (3)

21. The number of terms common to the two A.P.'s 3, 7, 11, ..., 407 and 2, 9, 16, ..., 709 is _____.

NTA Ans. (14)

ALLEN Ans. (14)

22. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=\sqrt{3}$, $|\vec{b}|=5$, $\vec{b}\cdot\vec{c}=10$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. If $\vec{a}$ is perpendicular to the vector $\vec{b}\times\vec{c}$, then $|\vec{a}\times(\vec{b}\times\vec{c})|$ is equal to _____.

NTA Ans. (30)

ALLEN Ans. (30)

23. If the distance between the plane, $23x - 10y - 2z + 48 = 0$ and the plane containing the lines $\frac{x+1}{2}=\frac{y-3}{4}=\frac{z+1}{3}$ and $\frac{x+3}{2}=\frac{y+2}{6}=\frac{z-1}{\lambda}(\lambda\in R)$ is equal to $\frac{k}{\sqrt{633}}$, then k is equal to _____.

NTA Ans. (3)

ALLEN Ans. (3)

24. If $C_r \equiv {}^{25}C_r$ and

$C_0 + 5.C_1 + 9.C_2 + ... + (101).C_{25} = 2^{25}.k$, then k is equal to _____.

NTA Ans. (51)

ALLEN Ans. (51)

25. If the curves, $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0$, $(k > 0)$ touch each other at a point, then the largest value of k is _____.

NTA Ans. (36)

ALLEN Ans. (36)