FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

(Held On Wednesday 08th JANUARY, 2020) TIME : 9 : 30 AM to 12 : 30 PM

MATHEMATICS

TEST PAPER WITH ANSWER

1. Let the line y = mx and the ellipse $2x^2 + y^2 = 1$ intersect at a ponit P in the first quadrant. If the normal to this ellipse at P meets the co-ordinate axes at $\left(-\frac{1}{3\sqrt{2}},0\right)$ and (0,β), then β is equal to

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{2\sqrt{2}}{3}$ (3) $\frac{2}{3}$ (4) $\frac{\sqrt{2}}{3}$

NTA Ans. (4)

ALLEN Ans. (4)

2. Let $f : R \to R$ be such that for all $x \in R$ $(2^{1+x} + 2^{1-x})$, $f(x)$ and $(3^x + 3^{-x})$ are in A.P., then the minimum value of $f(x)$ is

(1) 0 (2) 3 (3) 2 (4) 4

NTA Ans. (2)

ALLEN Ans. (2)

3. Let the volume of a parallelopiped whose coterminous edges are given by $\vec{u} = \hat{i} + \hat{j} + \lambda\hat{k}, \vec{v} = \hat{i} + \hat{j} + 3\hat{k}$ and $\vec{w} = 2\hat{i} + \hat{j} + \hat{k}$ be 1 cu. unit. If θ be the angle between the edges $\vec{u}$ and $\vec{w}$, then cosθ can be

(1) $\frac{7}{6\sqrt{3}}$ (2) $\frac{5}{7}$

(3) $\frac{7}{6\sqrt{6}}$ (4) $\frac{5}{3\sqrt{3}}$

NTA Ans. (1)

ALLEN Ans. (1)

4. If a,b and c are the greatest value of ${}^{19}C_p, {}^{20}C_q$ and ${}^{21}C_r$ respectively, then

(1) $\frac{a}{11} = \frac{b}{22} = \frac{c}{21}$ (2) $\frac{a}{10} = \frac{b}{11} = \frac{c}{21}$

(3) $\frac{a}{10} = \frac{b}{11} = \frac{c}{42}$ (4) $\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$

NTA Ans. (4)

ALLEN Ans. (4)

5. Let $f(x) = (\sin(\tan^{-1}x) + \sin(\cot^{-1}x))^2 - 1$, $|x| > 1$.

If $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\left(\sin^{-1}\left(f(x)\right)\right)$ and $y\left(\sqrt{3}\right) = \frac{\pi}{6}$, then $y\left(-\sqrt{3}\right)$ is equal to

(1) $\frac{5\pi}{6}$ (2) $-\frac{\pi}{6}$

(3) $\frac{\pi}{3}$ (4) $\frac{2\pi}{3}$

NTA Ans. (1)

ALLEN Ans. (BONUS)

6. $\lim\limits_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}$ is equal to

(1) $\frac{1}{e}$ (2) e^2

(3) e (4) $\frac{1}{e^2}$

NTA Ans. (4)

ALLEN Ans. (4)

7. Let two points be A(1,–1) and B(0,2). If a point P(x',y') be such that the area of ΔPAB = 5 sq. units and it lies on the line, 3x + y – 4λ = 0, then a value of λ is

(1) 1 (2) 4

(3) 3 (4) –3

NTA Ans. (3)

ALLEN Ans. (3)

8. The mean and the standard deviation (s.d.) of 10 observations are 20 and 2 resepectively. Each of these 10 observations is multiplied by p and then reduced by q, where $p \neq 0$ and $q \neq 0$. If the new mean and new s.d. become half of their original values, then q is equal to

(1) –20 (2) 10 (3) –10 (4) –5

NTA Ans. (1)

ALLEN Ans. (1)

9. Let y = y(x) be a solution of the differential equation, $\sqrt{1-x^2}\frac{dy}{dx}+\sqrt{1-y^2}=0, |x|<1$.

If $y\left(\frac{1}{2}\right)=\frac{\sqrt{3}}{2}$, then $y\left(\frac{-1}{\sqrt{2}}\right)$ is equal to

(1) $-\frac{\sqrt{3}}{2}$ (2) $\frac{1}{\sqrt{2}}$

(3) $\frac{\sqrt{3}}{2}$ (4) $-\frac{1}{\sqrt{2}}$

NTA Ans. (2)

ALLEN Ans. (BONUS)

10. If the equation, $x^2 + bx + 45 = 0$ ($b \in R$) has conjugate complex roots and they satisfy $|z+1| = 2\sqrt{10}$, then

(1) $b^2 - b = 42$

(2) $b^2 + b = 12$

(3) $b^2 + b = 72$

(4) $b^2 - b = 30$

NTA Ans. (4)

ALLEN Ans. (4)

11. For $a > 0$, let the curves $C_1 : y^2 = ax$ and $C_2 : x^2 = ay$ intersect at origin O and a point P. Let the line $x = b (0 < b < a)$ intersect the chord OP and the x-axis at points Q and R, respectively. If the line $x = b$ bisects the area bounded by the curves, C_1 and C_2, and the area of $\Delta OQR = \frac{1}{2}$, then 'a' satisfies the equation

(1) $x^6 - 12x^3 + 4 = 0$

(2) $x^6 - 12x^3 - 4 = 0$

(3) $x^6 + 6x^3 - 4 = 0$

(4) $x^6 - 6x^3 + 4 = 0$

NTA Ans. (1)

ALLEN Ans. (1)

12. Which one of the following is a tautology ?

(1) $P \wedge (P \vee Q)$

(2) $P \vee (P \wedge Q)$

(3) $Q \rightarrow (P \wedge (P \rightarrow Q))$

(4) $(P \wedge (P \rightarrow Q)) \rightarrow Q$

NTA Ans. (4)

ALLEN Ans. (4)

13. The locus of a point which divides the line segment joining the point (0,–1) and a point on the parabola, $x^2 = 4y$, internally in the ratio 1 : 2, is-

(1) $9x^2 - 3y = 2$ (2) $9x^2 - 12y = 8$

(3) $x^2 - 3y = 2$ (4) $4x^2 - 3y = 2$

NTA Ans. (2)

ALLEN Ans. (2)

14. If c is a point at which Rolle's theorem holds for the function, $f(x) = \log_e\left(\frac{x^2+\alpha}{7x}\right)$ in the interval [3,4], where $\alpha \in R$, then $f''(c)$ is equal to

(1) $\frac{\sqrt{3}}{7}$ (2) $\frac{1}{12}$

(3) $-\frac{1}{24}$ (4) $-\frac{1}{12}$

NTA Ans. (2)

ALLEN Ans. (2)

15. For which of the following ordered pairs (μ,δ), the system of linear equations

$x + 2y + 3z = 1$

$3x + 4y + 5z = \mu$

$4x + 4y + 4z = \delta$

is inconsistent ?

(1) (1,0) (2) (4,6)

(3) (3,4) (4) (4,3)

NTA Ans. (4)

ALLEN Ans. (4)

16. Let A and B be two independent events such that $P(A)=\frac{1}{3}$ and $P(B)=\frac{1}{6}$. Then, which of the following is TRUE ?

(1) $P(A/B)=\frac{2}{3}$

(2) $P(A/(A\cup B))=\frac{1}{4}$

(3) $P(A/B')=\frac{1}{3}$

(4) $P(A'/B')=\frac{1}{3}$

NTA Ans. (3)

ALLEN Ans. (3)

17. The inverse function of

$f(x)=\frac{8^{2x}-8^{-2x}}{8^{2x}+8^{-2x}}, x\in(-1,1)$, is

(1) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1-x}{1+x}\right)$

(2) $\frac{1}{4}\log_e\left(\frac{1-x}{1+x}\right)$

(3) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1+x}{1-x}\right)$

(4) $\frac{1}{4}\log_e\left(\frac{1+x}{1-x}\right)$

NTA Ans. (3)

ALLEN Ans. (3)

18. If $\int\frac{\cos x\,dx}{\sin^3 x\left(1+\sin^6 x\right)^{2/3}}=f(x)\left(1+\sin^6 x\right)^{1/\lambda}+c$

where c is a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$ is equal to

(1) –2 (2) $-\frac{9}{8}$ (3) 2 (4) $\frac{9}{8}$

NTA Ans. (1)

ALLEN Ans. (1)

19. The shortest distance between the lines

$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and

$\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ is

(1) $\frac{7}{2}\sqrt{30}$ (2) $3\sqrt{30}$ (3) 3 (4) $2\sqrt{30}$

NTA Ans. (2)

ALLEN Ans. (2)

20. Let $f(x) = x\cos^{-1}(-\sin|x|)$, $x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, then which of the following is true ?

(1) f' is decreasing in $\left(-\frac{\pi}{2},0\right)$ and increasing in $\left(0,\frac{\pi}{2}\right)$

(2) f is not differentiable at x = 0

(3) $f'(0)=-\frac{\pi}{2}$

(4) f' is increasing in $\left(-\frac{\pi}{2},0\right)$ and decreasing in $\left(0,\frac{\pi}{2}\right)$

NTA Ans. (1)

ALLEN Ans. (1)

21. The number of all 3 × 3 matrices A, with enteries from the set {–1,0,1} such that the sum of the diagonal elements of AA^T is 3, is

NTA Ans. (672.00)

ALLEN Ans. (672.00)

22. The least positive value of 'a' for which the equation $2x^2 + (a - 10)x + \frac{33}{2} = 2a$ has real roots is

NTA Ans. (8.00)

ALLEN Ans. (8.00)

23. Let the normal at a point P on the curve $y^2 - 3x^2 + y + 10 = 0$ intersect the y-axis at $\left(0, \frac{3}{2}\right)$. If m is the slope of the tangent at P to the curve, then $|m|$ is equal to

NTA Ans. (4.00)

ALLEN Ans. (4.00)

24. The sum $\sum_{k=1}^{20}(1+2+3+...+k)$ is ....

NTA Ans. (1540.00)

ALLEN Ans. (1540.00)

25. An urn contains 5 red marbles, 4 black marbles and 3 white marbles. Then the number of ways in which 4 marbles can be drawn so that at the most three of them are red is ....

NTA Ans. (490.00)

ALLEN Ans. (490.00 OR 13.00)