FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

(Held On Wednesday 08th JANUARY, 2020) TIME : 2 : 30 PM to 5 : 30 PM

MATHEMATICS

TEST PAPER WITH ANSWER

1. Let $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$ and $\vec{c} \cdot \vec{a} = 0$, then $\vec{c} \cdot \vec{b}$ is equal to

(1) $\frac{1}{2}$ (2) -1 (3) $-\frac{1}{2}$ (4) $-\frac{3}{2}$

NTA Ans. (3)

ALLEN Ans. (3)

2. The area (in sq. units) of the region $\{(x,y) \in R^2 : x^2 \le y \le 3 - 2x\}$, is

(1) $\frac{29}{3}$ (2) $\frac{31}{3}$ (3) $\frac{34}{3}$ (4) $\frac{32}{3}$

NTA Ans. (4)

ALLEN Ans. (4)

3. The length of the perpendicular from the origin, on the normal to the curve, $x^2 + 2xy - 3y^2 = 0$ at the point (2,2) is

(1) $4\sqrt{2}$ (2) $2\sqrt{2}$ (3) 2 (4) $\sqrt{2}$

NTA Ans. (2)

ALLEN Ans. (2)

4. If $I = \int_1^2 \frac{dx}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$, then :

(1) $\frac{1}{9} < I^2 < \frac{1}{8}$ (2) $\frac{1}{16} < I^2 < \frac{1}{9}$

(3) $\frac{1}{6} < I^2 < \frac{1}{2}$ (4) $\frac{1}{8} < I^2 < \frac{1}{4}$

NTA Ans. (1)

ALLEN Ans. (1)

5. If a line, y = mx + c is a tangent to the circle, $(x - 3)^2 + y^2 = 1$ and it is perpendicular to a line L_1, where L_1 is the tangent to the circle, $x^2 + y^2 = 1$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, then

(1) $c^2 - 6c + 7 = 0$

(2) $c^2 + 6c + 7 = 0$

(3) $c^2 + 7c + 6 = 0$

(4) $c^2 - 7c + 6 = 0$

NTA Ans. (2)

ALLEN Ans. (2)

6. Let S be the set of all functions $f : [0,1] \to R$, which are continuous on [0,1] and differentiable on (0,1). Then for every f in S, there exists a $c \in (0,1)$, depending on f, such that

(1) $|f(c) - f(1)| < (1 - c)|f'(c)|$

(2) $|f(c) - f(1)| < |f'(c)|$

(3) $|f(c) + f(1)| < (1 + c)|f'(c)|$

(4) $\frac{f(1) - f(c)}{1 - c} = f'(c)$

NTA Ans. (2)

ALLEN Ans. (BONUS)

7. Which of the following statements is a tautology?

(1) $\sim(p \vee \sim q) \to p \vee q$

(2) $\sim(p \wedge \sim q) \to p \vee q$

(3) $\sim(p \vee \sim q) \to p \wedge q$

(4) $p \vee (\sim q) \to p \wedge q$

NTA Ans. (1)

ALLEN Ans. (1)

8. If the 10^{th} term of an A.P. is $\frac{1}{20}$ and its 20^{th} term is $\frac{1}{10}$, then the sum of its first 200 terms is

(1) $50\frac{1}{4}$ (2) $100\frac{1}{2}$

(3) 50 (4) 100

NTA Ans. (2)

ALLEN Ans. (2)

9. Let $f : (1,3) \rightarrow R$ be a function defined by $f(x) = \frac{x[x]}{1+x^2}$, where [x] denotes the greatest integer $\le$ x. Then the range of f is

(1) $\left(\frac{3}{5},\frac{4}{5}\right)$ (2) $\left(\frac{2}{5},\frac{3}{5}\right] \cup \left(\frac{3}{4},\frac{4}{5}\right)$

(3) $\left(\frac{2}{5},\frac{4}{5}\right]$ (4) $\left(\frac{2}{5},\frac{1}{2}\right) \cup \left(\frac{3}{5},\frac{4}{5}\right]$

NTA Ans. (4)

ALLEN Ans. (4)

10. The system of linear equations

$\lambda x + 2y + 2z = 5$

$2\lambda x + 3y + 5z = 8$

$4x + \lambda y + 6z = 10$ has

(1) infinitely many solutions when $\lambda = 2$

(2) a unique solution when $\lambda = -8$

(3) no solution when $\lambda = 8$

(4) no solution when $\lambda = 2$

NTA Ans. (4)

ALLEN Ans. (4)

11. If α and β be the coefficients of x^4 and x^2 respectively in the expansion of

$\left(x+\sqrt{x^2-1}\right)^6 + \left(x-\sqrt{x^2-1}\right)^6$, then

(1) $\alpha + \beta = 60$ (2) $\alpha + \beta = -30$

(3) $\alpha - \beta = -132$ (4) $\alpha - \beta = 60$

NTA Ans. (3)

ALLEN Ans. (3)

12. $\lim_{x\to 0} \frac{\int_0^x t\sin(10t)\,dt}{x}$ is equal to

(1) 0 (2) $-\frac{1}{5}$

(3) $-\frac{1}{10}$ (4) $\frac{1}{10}$

NTA Ans. (1)

ALLEN Ans. (1)

13. If $A = \begin{pmatrix} 2 & 2 \\ 9 & 4 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then $10A^{-1}$ is equal to

(1) 4I – A (2) A – 6I

(3) 6I – A (4) A – 4I

NTA Ans. (2)

ALLEN Ans. (2)

14. The mean and variance of 20 observations are found to be 10 and 4, respectively. On rechecking, it was found that an observation 9 was incorrect and the correct observation was 11. Then the correct variance is

(1) 3.99 (2) 3.98

(3) 4.02 (4) 4.01

NTA Ans. (1)

ALLEN Ans. (1)

15. If a hyperbola passes through the point P(10,16) and it has vertices at $(\pm 6,0)$, then the equation of the normal to it at P is

(1) $x + 2y = 42$ (2) $3x + 4y = 94$

(3) $2x + 5y = 100$ (4) $x + 3y = 58$

NTA Ans. (3)

ALLEN Ans. (3)

16. Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability that A or B occurs is $\frac{1}{2}$, then the probability of both of them occur together is

(1) 0.02 (2) 0.01 (3) 0.20 (4) 0.10

NTA Ans. (4)

ALLEN Ans. (4)

17. The mirror image of the point (1,2,3) in a plane is $\left(-\frac{7}{3},-\frac{4}{3},-\frac{1}{3}\right)$. Which of the following points lies on this plane ?

(1) $(-1, -1, -1)$ (2) $(-1, -1, 1)$

(3) $(1, 1, 1)$ (4) $(1, -1, 1)$

NTA Ans. (4)

ALLEN Ans. (4)

18. Let S be the set of all real roots of the equation, $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$. Then S :

(1) is an empty set.

(2) contains at least four elements.

(3) contains exactly two elements.

(4) is a singleton.

NTA Ans. (4)

ALLEN Ans. (4)

19. Let $\alpha = \frac{-1+i\sqrt{3}}{2}$. If $a = (1+\alpha)\sum_{k=0}^{100}\alpha^{2k}$ and $b = \sum_{k=0}^{100}\alpha^{3k}$, then a and b are the roots of the quadratic equation :

(1) $x^2 - 102x + 101 = 0$

(2) $x^2 + 101x + 100 = 0$

(3) $x^2 - 101x + 100 = 0$

(4) $x^2 + 102x + 101 = 0$

NTA Ans. (1)

ALLEN Ans. (1)

20. The differential equation of the family of curves, $x^2 = 4b(y + b)$, $b \in R$, is

(1) $x(y')^2 = x + 2yy'$

(2) $x(y')^2 = 2yy' - x$

(3) $xy'' = y'$

(4) $x(y')^2 = x - 2yy'$

NTA Ans. (1)

ALLEN Ans. (1)

21. If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$, $\alpha, \beta \in \left(0,\frac{\pi}{2}\right)$, then $\tan(\alpha + 2\beta)$ is equal to _____.

NTA Ans. (1)

ALLEN Ans. (1.00)

22. Let $f(x)$ be a polynomial of degree 3 such that $f(-1) = 10$, $f(1) = -6$, $f(x)$ has a critical point at $x = -1$ and $f'(x)$ has a critical point at $x = 1$. Then $f(x)$ has a local minima at x = _______.

NTA Ans. (3)

ALLEN Ans. (3.00)

23. Let a line $y = mx$ $(m > 0)$ intersect the parabola, $y^2 = x$ at a point P, other than the origin. Let the tangent to it at P meet the x-axis at the point Q. If area $(\Delta OPQ) = 4$ sq. units, then m is equal to __________.

NTA Ans. (0.50)

ALLEN Ans. (0.50)

24. The sum, $\sum_{n=1}^{7}\frac{n(n+1)(2n+1)}{4}$ is equal to ________.

NTA Ans. (504)

ALLEN Ans. (504.00)

25. The number of 4 letter words (with or without meaning) that can be formed from the eleven letters of the word 'EXAMINATION' is _______.

NTA Ans. (2454)

ALLEN Ans. (2454.00)