FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

(Held On Tuesday 07th JANUARY, 2020) TIME : 9 : 30 AM to 12 : 30 PM

MATHEMATICS

TEST PAPER WITH ANSWER

1. If $g(x) = x^2 + x - 1$ and

$(gof)(x) = 4x^2 - 10x + 5$, then $f\left(\frac{5}{4}\right)$ is equal to

(1) $\frac{3}{2}$ (2) $-\frac{1}{2}$ (3) $-\frac{3}{2}$ (4) $\frac{1}{2}$

NTA Ans. (2)

ALLEN Ans. (2)

2. If $\text{Re}\left(\frac{z-1}{2z+i}\right) = 1$, where $z = x + iy$, then the point (x,y) lies on a :

(1) circle whose centre is at $\left(-\frac{1}{2}, -\frac{3}{2}\right)$

(2) circle whose diameter is $\frac{\sqrt{5}}{2}$

(3) straight line whose slope is $\frac{3}{2}$

(4) straight line whose slope is $-\frac{2}{3}$

NTA Ans. (2)

ALLEN Ans. (2)

3. Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is :

(1) $\frac{21}{2}$ (2) 27 (3) 16 (4) 7

NTA Ans. (3)

ALLEN Ans. (3)

4. If

$$y(\alpha) = \sqrt{2\left(\frac{\tan\alpha + \cot\alpha}{1+\tan^2\alpha}\right) + \frac{1}{\sin^2\alpha}}, \alpha \in \left(\frac{3\pi}{4}, \pi\right),$$

then $\frac{dy}{d\alpha}$ at $\alpha = \frac{5\pi}{6}$ is :

(1) 4 (2) $-\frac{1}{4}$ (3) $\frac{4}{3}$ (4) –4

NTA Ans. (1)

ALLEN Ans. (1)

5. Let α be a root of the equation $x^2 + x + 1 = 0$ and the matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}$, then the matrix A^{31} is equal to:

(1) A^3 (2) A (3) A^2 (4) I_3

NTA Ans. (1)

ALLEN Ans. (1)

6. If $y = mx + 4$ is a tangent to both the parabolas, $y^2 = 4x$ and $x^2 = 2by$, then b is equal to :

(1) 128 (2) –64 (3) –128 (4) –32

NTA Ans. (3)

ALLEN Ans. (3)

7. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is :

(1) $\sqrt{3}$ (2) $2\sqrt{3}$ (3) $3\sqrt{2}$ (4) $\frac{3}{\sqrt{2}}$

NTA Ans. (3)

ALLEN Ans. (3)

8. An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value k when k consecutive heads are obtained for k = 3, 4, 5 otherwise X takes the value –1. Then the expected value of X, is :

(1) $\frac{3}{16}$ (2) $-\frac{3}{16}$ (3) $\frac{1}{8}$ (4) $-\frac{1}{8}$

NTA Ans. (3)

ALLEN Ans. (3)

9. The area of the region, enclosed by the circle $x^2 + y^2 = 2$ which is not common to the region bounded by the parabola $y^2 = x$ and the straight line y = x, is :

(1) $\frac{1}{3}(12\pi - 1)$ (2) $\frac{1}{6}(12\pi - 1)$

(3) $\frac{1}{6}(24\pi - 1)$ (4) $\frac{1}{3}(6\pi - 1)$

NTA Ans. (2)

ALLEN Ans. (2)

10. Let $x^k + y^k = a^k$, (a, K > 0) and $\frac{dy}{dx} + \left(\frac{y}{x}\right)^{\frac{1}{3}} = 0$, then k is :

(1) $\frac{3}{2}$ (2) $\frac{1}{3}$ (3) $\frac{2}{3}$ (4) $\frac{4}{3}$

NTA Ans. (3)

ALLEN Ans. (3)

11. If y = y(x) is the solution of the differential equation, $e^y\left(\frac{dy}{dx} - 1\right) = e^x$ such that $y(0) = 0$, then y(1) is equal to :

(1) $2 + \log_e 2$ (2) $2e$

(3) $\log_e 2$ (4) $1 + \log_e 2$

NTA Ans. (4)

ALLEN Ans. (4)

12. Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 appear, is :

(1) $\frac{5}{2}(6!)$ (2) 5^6 (3) $\frac{1}{2}(6!)$ (4) 6!

NTA Ans. (1)

ALLEN Ans. (1)

13. Let P be a plane passing through the points (2, 1, 0), (4, 1, 1) and (5, 0, 1) and R be any point (2, 1, 6). Then the image of R in the plane P is :

(1) (6, 5, –2) (2) (4, 3, 2)

(3) (3, 4, –2) (4) (6, 5, 2)

NTA Ans. (1)

ALLEN Ans. (1)

14. A vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}(\alpha, \beta \in R)$ lies in the plane of the vectros $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$. If $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$, then:

(1) $\vec{a}.\hat{i} + 1 = 0$ (2) $\vec{a}.\hat{i} + 3 = 0$

(3) $\vec{a}.\hat{k} + 4 = 0$ (4) $\vec{a}.\hat{k} + 2 = 0$

NTA Ans. (4)

ALLEN Ans. (BONUS)

15. If $f(a + b + 1 - x) = f(x)$, for all x, where a and b are fixed positive real numbers, then $\frac{1}{a+b}\int_a^b x(f(x) + f(x+1))dx$ is equal to :

(1) $\int_{a+1}^{b+1} f(x)dx$ (2) $\int_{a+1}^{b+1} f(x+1)dx$

(3) $\int_{a-1}^{b-1} f(x+1)dx$ (4) $\int_{a-1}^{b-1} f(x)dx$

NTA Ans. (1)

ALLEN Ans. (1 OR 3)

16. Let the function, $f : [-7, 0] \to R$ be continuous on $[-7, 0]$ and differentiable on $(-7, 0)$. If $f(-7) = -3$ and $f'(x) \le 2$, for all $x \in (-7, 0)$, then for all such functions f, $f(-1) + f(0)$ lies in the interval :

(1) $[-6, 20]$ (2) $(-\infty, 20]$

(3) $(-\infty, 11]$ (4) $[-3, 11]$

NTA Ans. (2)

ALLEN Ans. (2)

17. If the system of linear equations

$2x + 2ay + az = 0$

$2x + 3by + bz = 0$

$2x + 4cy + cz = 0$,

where $a, b, c \in R$ are non-zero and distinct; has a non-zero solution, then :

(1) a, b, c are in A.P.

(2) $a + b + c = 0$

(3) a, b, c are in G.P.

(4) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

NTA Ans. (4)

ALLEN Ans. (4)

18. Let α and β be two real roots of the equation $(k + 1)\tan^2 x - \sqrt{2} \cdot \lambda \tan x = (1 - k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha + \beta) = 50$, then a value of λ is ;

(1) 5 (2) 10 (3) $5\sqrt{2}$ (4) $10\sqrt{2}$

NTA Ans. (2)

ALLEN Ans. (2)

19. The logical statement $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to :

(1) p (2) q (3) $\sim p$ (4) $\sim q$

NTA Ans. (3)

ALLEN Ans. (3)

20. The greatest positive integer k, fr which $49^k + 1$ is a factor of the sum

$49^{125} + 49^{124} + \ldots 49^2 + 49 + 1$, is :

(1) 32 (2) 60 (3) 63 (4) 65

NTA Ans. (3)

ALLEN Ans. (3)

21. $\lim\limits_{x \to 2} \frac{3^x + 3^{3-x} - 12}{3^{-x/2} - 3^{1-x}}$ is equal to _____.

NTA Ans. (36)

ALLEN Ans. (36.00)

22. If the variance of the first n natural numbers is 10 and the variance of the first m even natural numbers is 16, then m + n is equal to _____.

NTA Ans. (18)

ALLEN Ans. (18.00)

23. If the sum of the coefficients of all even powers of x in the product

$(1 + x + x^2 + \ldots + x^{2n})(1 - x + x^2 - x^3 + \ldots + x^{2n})$ is 61, then n is equal to _____.

NTA Ans. (30)

ALLEN Ans. (30.00)

24. Let A(1, 0), B(6, 2) and $C\left(\frac{3}{2}, 6\right)$ be the vertices of a triangle ABC. If P is a point inside the triangle ABC such that the triangles APC, APB and BPC have equal areas, then the length of the line segment PQ, where Q is the point $\left(-\frac{7}{6}, -\frac{1}{3}\right)$, is _____.

NTA Ans. (5)

ALLEN Ans. (5.00)

25. Let S be the set of points where the function, $f(x) = \left|2 - |x - 3|\right|$, $x \in R$, is not differentiable.

Then $\sum\limits_{x \in S} f(f(x))$ is equal to _____.

NTA Ans. (3)

ALLEN Ans. (3.00)