FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

(Held On Tuesday 07th JANUARY, 2020) TIME : 2 : 30 PM to 5 : 30 PM

MATHEMATICS

TEST PAPER WITH ANSWER

1. Let $y = y(x)$ be a function of x satisfying $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ where k is a constant and $y\left(\frac{1}{2}\right) = -\frac{1}{4}$. Then $\frac{dy}{dx}$ at $x = \frac{1}{2}$, is equal to:

(1) $\frac{\sqrt{5}}{2}$ (2) $-\frac{\sqrt{5}}{2}$

(3) $\frac{2}{\sqrt{5}}$ (4) $-\frac{\sqrt{5}}{4}$

NTA Ans. (2)

ALLEN Ans. (2)

2. The area (in sq. units) of the region $\{(x, y) \in R^2 | 4x^2 \le y \le 8x + 12)$ is :

(1) $\frac{127}{3}$ (2) $\frac{125}{3}$ (3) $\frac{124}{3}$ (4) $\frac{128}{3}$

NTA Ans. (4)

ALLEN Ans. (4)

3. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three units vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. If $\lambda = \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ and $\vec{d}=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$, then the ordered pair, $(\lambda,\vec{d})$ is equal to :

(1) $\left(-\frac{3}{2},3\vec{a}\times\vec{b}\right)$ (2) $\left(-\frac{3}{2},3\vec{c}\times\vec{b}\right)$

(3) $\left(\frac{3}{2},3\vec{b}\times\vec{c}\right)$ (4) $\left(\frac{3}{2},3\vec{a}\times\vec{c}\right)$

NTA Ans. (1)

ALLEN Ans. (1)

4. If the sum of the first 40 terms of the series, $3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + \ldots$ is $(102)m$, then m is equal to :

(1) 20 (2) 5 (3) 10 (4) 25

NTA Ans. (1)

ALLEN Ans. (1)

5. The value of c in the Lagrange's mean value theorem for the function $f(x) = x^3 - 4x^2 + 8x + 11$, when $x \in [0, 1]$ is :

(1) $\frac{2}{3}$ (2) $\frac{\sqrt{7}-2}{3}$

(3) $\frac{4-\sqrt{5}}{3}$ (4) $\frac{4-\sqrt{7}}{3}$

NTA Ans. (4)

ALLEN Ans. (4)

6. If θ_1 and θ_2 be respectively the smallest and the largest values of θ in $(0, 2\pi) - \{\pi\}$ which satisfy the equation, $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$, then $\int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta$ is equal to :

(1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{3}+\frac{1}{6}$ (3) $\frac{\pi}{9}$ (4) $\frac{\pi}{3}$

NTA Ans. (4)

ALLEN Ans. (4)

7. The number of ordered pairs (r, k) for which $6\cdot{}^{35}C_r = (k^2 - 3)\cdot{}^{36}C_{r+1}$, where k is an integer, is :

(1) 3 (2) 2 (3) 4 (4) 6

NTA Ans. (3)

ALLEN Ans. (3)

8. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3×3 real matrices such that $b_{ij} = (3)^{(i+j-2)}a_{ji}$, where i, j = 1, 2, 3. If the determinant of B is 81, then the determinant of A is :

(1) 3 (2) 1/3 (3) 1/81 (4) 1/9

NTA Ans. (4)

ALLEN Ans. (4)

9. Let $a_1, a_2, a_3, \ldots$ be a G.P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$, then λ is equal to :

(1) -171 (2) 171 (3) $\frac{511}{3}$ (4) -513

NTA Ans. (1)
ALLEN Ans. (1)

10. Let A, B, C and D be four non-empty sets. The contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$, then $A \subseteq C$" is :

(1) If $A \subseteq C$, then $B \subset A$ or $D \subset B$

(2) If $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$

(3) If $A \not\subseteq C$, then $A \subseteq B$ and $B \subseteq D$

(4) If $A \not\subseteq C$, then $A \not\subseteq B$ and $B \subseteq D$

NTA Ans. (2)
ALLEN Ans. (2)

11. If $3x + 4y = 12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ for some $a \in R$, then the distance between the foci of the ellipse is :

(1) 4 (2) $2\sqrt{7}$ (3) $2\sqrt{5}$ (4) $2\sqrt{2}$

NTA Ans. (2)
ALLEN Ans. (2)

12. The value of α for which $4\alpha\int_{-1}^{2} e^{-\alpha|x|}dx = 5$, is :

(1) $\log_e\left(\frac{3}{2}\right)$ (2) $\log_e\left(\frac{4}{3}\right)$

(3) $\log_e 2$ (4) $\log_e\sqrt{2}$

NTA Ans. (3)
ALLEN Ans. (3)

13. The coefficient of x^7 in the expression $(1 + x)^{10} + x(1 + x)^9 + x^2(1 + x)^8 + \ldots + x^{10}$ is :

(1) 120 (2) 330 (3) 210 (4) 420

NTA Ans. (2)
ALLEN Ans. (2)

14. Let α and β be the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k$, $k \geq 1$, then which one of the following statements is not true ?

(1) $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$

(2) $p_5 = 11$

(3) $p_3 = p_5 - p_4$

(4) $p_5 = p_2 \cdot p_3$

NTA Ans. (4)
ALLEN Ans. (4)

15. The locus of the mid-points of the perpendiculars drawn from points on the line, $x = 2y$ to the line $x = y$ is :

(1) $2x - 3y = 0$ (2) $7x - 5y = 0$
(3) $5x - 7y = 0$ (4) $3x - 2y = 0$

NTA Ans. (3)
ALLEN Ans. (3)

16. If $\frac{3 + i\sin\theta}{4 - i\cos\theta}, \theta \in [0, 2\pi]$, is a real number, then an argument of $\sin\theta + i\cos\theta$ is :

(1) $-\tan^{-1}\left(\frac{3}{4}\right)$ (2) $\tan^{-1}\left(\frac{4}{3}\right)$

(3) $\pi - \tan^{-1}\left(\frac{4}{3}\right)$ (4) $\pi - \tan^{-1}\left(\frac{3}{4}\right)$

NTA Ans. (3)
ALLEN Ans. (3)

17. Let $y = y(x)$ be the solution curve of the differential equation, $(y^2 - x)\frac{dy}{dx} = 1$, satisfying $y(0) = 1$. This curve intersects the x-axis at a point whose abscissa is :

(1) $2 + e$ (2) 2 (3) $2 - e$ (4) $-e$

NTA Ans. (3)
ALLEN Ans. (3)

18. Let f(x) be a polynomial of degree 5 such that $x = \pm 1$ are its critical points. If $\lim\limits_{x\to 0}\left(2+\frac{f(x)}{x^3}\right)=4$, then which one of the following is not true?

(1) f is an odd function

(2) x = 1 is a point of minima and x = –1 is a point of maxima of f.

(3) x = 1 is a point of maxima and x = –1 is a point of minimum of f.

(4) f(1) – 4f(–1) = 4

NTA Ans. (2)

ALLEN Ans. (2)

19. In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3 k$, then k is equal to :

(1) $\frac{17}{2}$ (2) 4 (3) $\frac{17}{8}$ (4) $\frac{17}{4}$

NTA Ans. (3)

ALLEN Ans. (3)

20. Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points A and B. The $(AB)^2$ is equal to :

(1) $\frac{52}{5}$ (2) $\frac{32}{5}$ (3) $\frac{56}{5}$ (4) $\frac{64}{5}$

NTA Ans. (4)

ALLEN Ans. (4)

21. If the system of linear equations,

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$3x + 2y + \lambda z = \mu$$

has more two solutions, then $\mu - \lambda^2$ is equal to _____

NTA Ans. (13.00)

ALLEN Ans. (13.00)

22. If the function f defined on $\left(-\frac{1}{3},\frac{1}{3}\right)$ by

$$f(x)=\begin{cases}\frac{1}{x}\log_e\left(\frac{1+3x}{1-2x}\right) & ,\text{when } x\neq 0\\ k & ,\text{when } x=0\end{cases}$$

is continuous, then k is equal to_____

NTA Ans. (5.00)

ALLEN Ans. (5.00)

23. If the mean and variance of eight numbers 3, 7, 9, 12, 13, 20, x and y be 10 and 25 respectively, then x·y is equal to_____

NTA Ans. (54.00)

ALLEN Ans. (54.00)

24. If the foot of the perpendicular drawn from the point (1, 0, 3) on a line passing through $(\alpha, 7, 1)$ is $\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$, then α is equal to_____

NTA Ans. (4.00)

ALLEN Ans. (4.00)

25. Let $X = \{n \in N : 1 \le n \le 50\}$. If

$A = \{n \in X : n \text{ is a multiple of } 2\}$ and

$B = \{n \in X : n \text{ is a multiple of } 7\}$, then the number of elements in the smallest subset of X containing both A and B is_____

NTA Ans. (29.00)

ALLEN Ans. (29.00)